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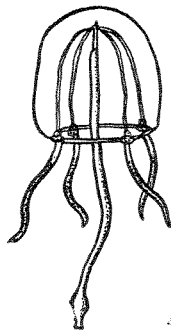
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**TIDAL CURRENT CHOKING IN THE
LANDLOCKED FJORD OF NORDÅSVATNET**

By

B. GLENNE and T. SIMENSEN

Norwegian Institute for Water Research, Blindern



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ABSTRACT

Due to heavy pollutional loading of the landlocked fjord Nordåsvatnet, situated south of Bergen, the interchange of water through the narrow canal between Nordåsvatnet and the open fjord, has been surveyed. Tidal heights were measured outside Straume and at three different locations in the landlocked fjord. Hydraulic information about Straume canal were made available by means of velocity measurements over a tidal period. A mathematical formulation of the interchange process, supplied with constants from the observational data was fitted on to an analog computer. In this way canal velocity, canal flow and tidal elevation in Nordåsvatnet were calculated for a varying canal area.

List of symbols

A — cross-sectional area of canal at Straume.	n — Manning's roughness coefficient.
A_N — effective surface area of Nordåsvatnet and Sælevatn.	q — rate of flow through the canal at Straume.
f — phase angle.	q_f — freshwater inflow rate.
H_B — tidal amplitude in Bergen.	R — hydraulic radius.
H_u — tidal amplitude outside Straume canal.	S — slope of energy line.
h_i — tidal elevation in Nordåsvatnet.	T — tidal period.
h_u — tidal elevation outside Straume canal.	t — time.
$\Delta h = h_u - h_i$.	V — average velocity of the water in the Straume.
k — constant.	ω — angular frequency.
k_1 — constant.	π — 3.1415
L — average length of canal at Straume.	

INTRODUCTION

GENERAL STATEMENT OF THE PROBLEM

It is the specific purpose of this investigation to study the tidal interchange of water in Nordåsvatnet (Nordåsvannet).

To carry this out a basic review and investigation of the hydrographic conditions in Nordåsvatnet is necessary. This report will therefore not limit itself to only the tidal choking effect in a landlocked fjord caused by a narrow inlet, but will also attempt to serve as a base and stepping-stone for further investigations.

SCOPE OF THE INVESTIGATION

Fana, situated just south of Bergen, is today a rapidly growing community consisting of mainly residential areas. With Nordåsvatnet as the only natural recipient of sewage, the pollutional load on this landlocked fjord has become a major concern to the local authorities. The unesthetic appearance and often offensive odour of the fjord seriously detracts from the otherwise beautiful scenery.

The Norwegian Institute for Water Research was in 1959 consulted by the local authorities in Fana to determine the degree of pollution and the possible remedies for controlling its growth. As a first step these authorities in March 1960 arranged, on request from the Institute, a meeting of the leading authorities in the field in order to establish and summarize what is known about the hydrographical and biological condition of Nordåsvatnet. The meeting gave a valuable background for further investigations and pointed out the most important problems to be solved. Among these problems was the determination of the replacement of brackish water as a function of the tidal interchange of water through the narrow canal between Nordåsvatnet and the open fjord.

This study has used published material and data to compute drainage areas, volumes, precipitation, evapo-transpiration, freshwater inflow, circulation etc. When combined with unpublished data and observations made in the winter of 1960/61 the present hydrographical state of Nordåsvatnet can be outlined.

For investigation of the choking effect extensive data were collected. Tidal heights were measured outside Straume (Strømme) and at three different locations in the fjord. In Feb./March 1961 tidal heights were measured by automatic recorders inside and outside the Straume canal. To complete the hydraulic information about Straume canal, velocities, streamflow and salinities were measured. Some of the currents in the fjord itself were determined by placing a suspension of the *Serratia indica* bacterium in the Straume canal and later retrieving the bacteria at different places in the fjord.

These data were used to form and verify a theory which makes one able to predict flow in the canal at Straume and tidal elevations within the fjord.

REVIEW OF PREVIOUS WORK

In Scandinavia the characteristic long and narrow landlocked fjords with their so-called thresholds are often surrounded by densely populated areas. Stockholm and Oslo exemplify this and both cities are greatly concerned with pollution in their landlocked fjord.

It is therefore not surprising to find in Scandinavia a considerable amount of literature and data published on the subject of landlocked fjords.

Many of the distinctive landlocked fjords are located on the west coast of Norway and it was here GAARDER (1916, 1919) in 1912 started his investigations. GAARDER performed extensive oxygen and alkalinity measurements in several fjords, among them Nordåsvatnet and Dolviken (Fig. 1). His data and conclusions have later become an almost standard reference. From the oxygen and alkalinity data GAARDER attempted to define the circulations and interchanges of water in Nordåsvatnet. In the outer basin of Nordåsvatnet (Fig. 2) GAARDER showed that there exists a complete yearly interchange of the water. Under favourable meteorological conditions this interchange may take place more often. In the inner basin (Fig. 2) GAARDER observed one complete interchange, but this was more of an exception than a predictable annual event. Although GAARDER has classified the different interchanges and circulations it appears that the mechanisms are irregular and very sensitive to meteorological parameters such as wind, temperature and precipitation. In the summers of 1932 and 1933 STRÖM (1936) investigated thirty landlocked fjords on the west coast of Norway. Salinity, temperature, oxygen, pH and phosphate measurements were made in Nordåsvatnet. STRÖM's interest was, however, mainly confined to the sediments at the deep parts of the lake.

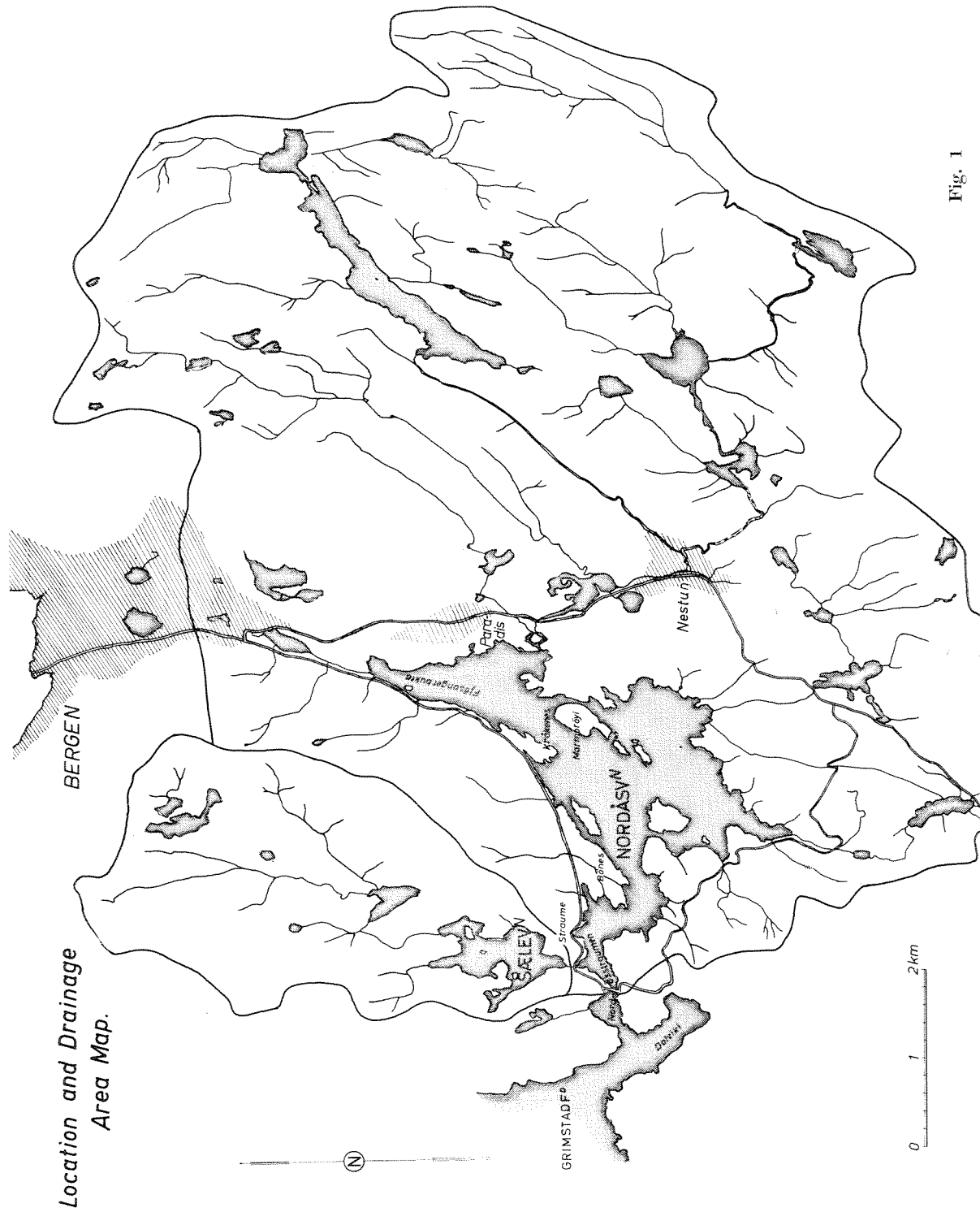
From May 1941 to July 1942 temperature, oxygen, salinity and plankton data were collected by WIBORG (1944). WIBORG continued GAARDER's extensive discussion and mentions the same complete interchange of the water in the outer basin. In the inner basin WIBORG found in the spring of 1941 a partial renewal of the deep water, while during the winter and spring of 1942 a complete interchange of the water took place. The circulation and renewal of the surface water was found to depend greatly on the meteorological factors. A description of the standing crop of plankton composes the main part of WIBORG's article. VINDENES (1944) has published some tidal measurements taken at Kråkenes (Fig. 2). No comparison is made with the tides outside Straume, although he tried to show the influence of wind and precipitation. VINDENES' measurements were made before the cross-sectional area of the canal at Straume (Fig. 5) was enlarged in 1955.

MOSBY and VOGELSANG (1949) conducted in 1948 a bacteriological investigation of the water in Nordåsvatnet. The surface water in the inner basin was found to be polluted. However, below a depth of two metres and down to the discontinuity layer of density the water was comparatively pure. In the outer basin, even the surface water was found relatively unpolluted. Besides performing plate count and coli bacteria determination MOSBY and VOGELSANG also measured salinity and temperature.

WIBORG's data (1944) were used by BRAARUD and HOPE (1952) to find the annual variation in phytoplankton in Nordåsvatnet. From March 1956 to February 1958 P. A. SOLHEIM measured salinity and temperature in Sælevatn (Fig. 2). SOLHEIM's data are unpublished but of importance since they show the vertical motion of the discontinuity layer of density in Sælevatn.

Finally in direct connection with Nordåsvatnet must be mentioned the conference which took place in Bergen 18 March 1960. Leading authorities were invited to a round table discussion where the pollution problem in Nordåsvatnet was discussed.¹

¹ At the time of writing, the manuscript of this conference has not yet been published.



Location and Drainage Area Map.

Fig. 1

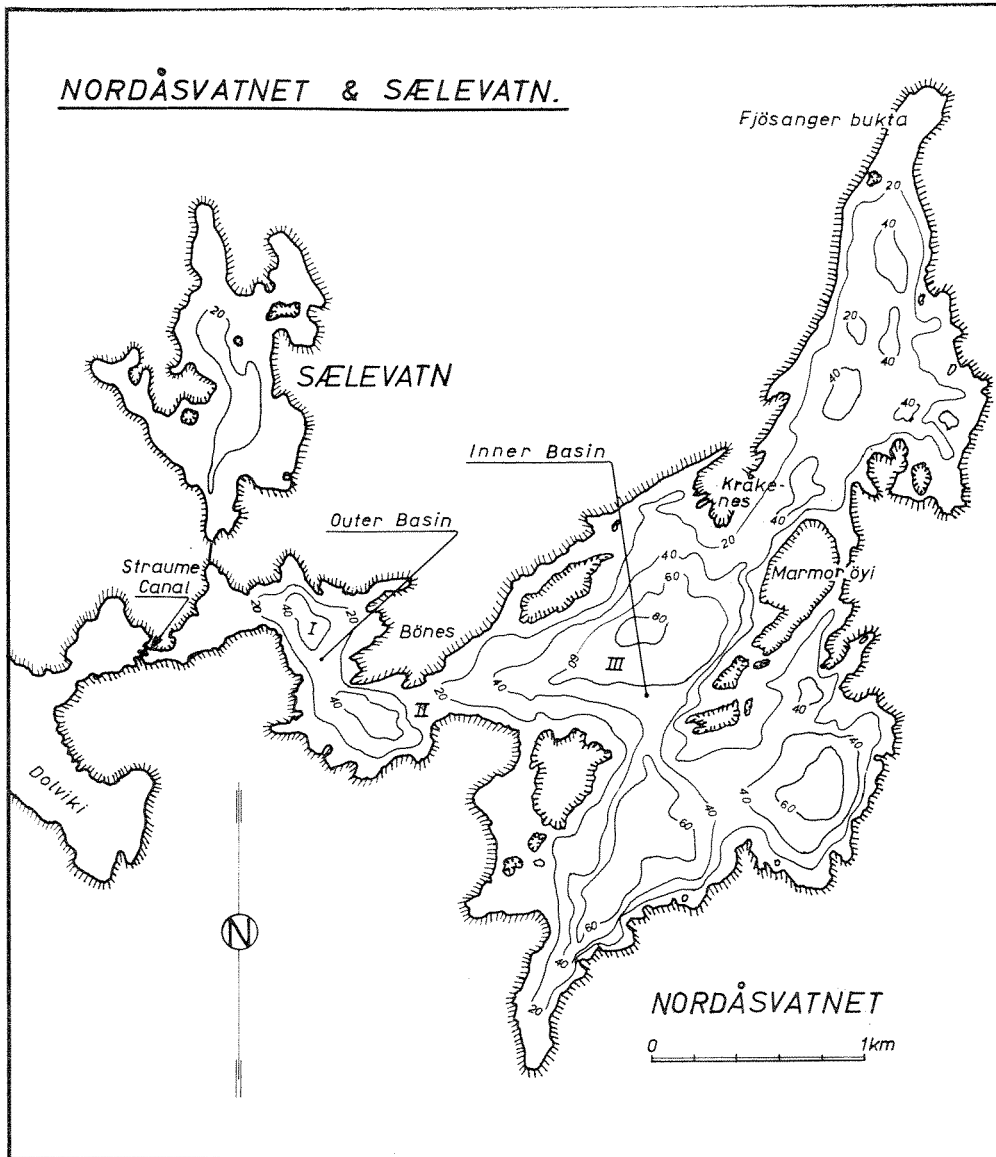


Fig. 2.

The city of Stockholm has had a somewhat similar problem to that of Nordåsvatnet, since its harbour water has gradually become more polluted. In 1945 PETERSON (1946) supervised the investigations of Stockholm harbour and recommended a partial lowering of the threshold at Oxdjupet. KULLENBERG, (1946, 1949 and undated report), who carried out the hydraulic computations, shows a 35% increase in the inflow of water to Stockholm Harbour after the threshold was lowered. A simultaneous inflow and outflow of water exists here across the threshold since the tidal action is virtually nil.

The influence of meteorological parameters upon the temperature and salinity of the fjord waters, south of Nordåsvatnet, has been observed by TAMBS-LYCHE (1957).

CALDWELL (1955) reviews the most important inlets along the coasts of the U.S.A., and classifies them into three groups. Landlocked fjords fit into group three with the velocity across the threshold expressed as the square root of the inside and outside tidal height difference. EINSTEIN and FUCHS (1955) develop the common differential equations connected with tidal flow in inlets and show methods of solving the equations. A more theoretical approach to the problem is offered by DRONKERS and SCHÖNFELD (1955), who also mention a means of solving the equations by using electronic computers. A digital computer is used by OTTER and DAY (1960) to solve the problem of tidal heights and velocities in rivers and estuaries. OTTER and DAY let the width of the estuary vary exponentially with its length, and express the friction as a linear function of the current velocity. ROSE (1961) uses a digital computer (an IBM 650) to solve the tidal problem in the North Sea and its estuaries.

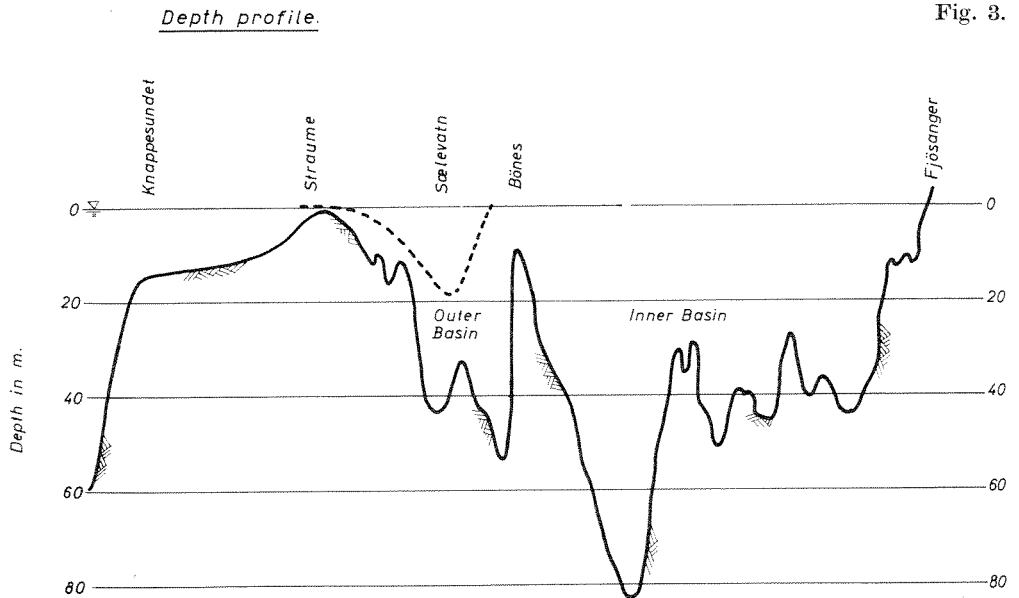
A short summary of analog computers and of "Mons", the machine used for this study, is given by RUGE (1961).

TIDAL CHOKING

GOVERNING PARAMETERS

Topography

Nordåsvatnet, at $60^{\circ}19'N$ and $5^{\circ}19'E$, is located approximately 7 kilometres south of Bergen (Fig. 1). It consists of an outer and an inner basin separated by a submerged ridge where the depth is approximately 10 metres (Fig. 3). The outer basin (west of Bønes) has a maximum depth of 53 metres. The inner basin has a maximum depth of 84 metres, and a broken-up topography. The inner basin is the larger one with 88% of the surface area of Nordåsvatnet and 92% of the total volume (Fig. 4).



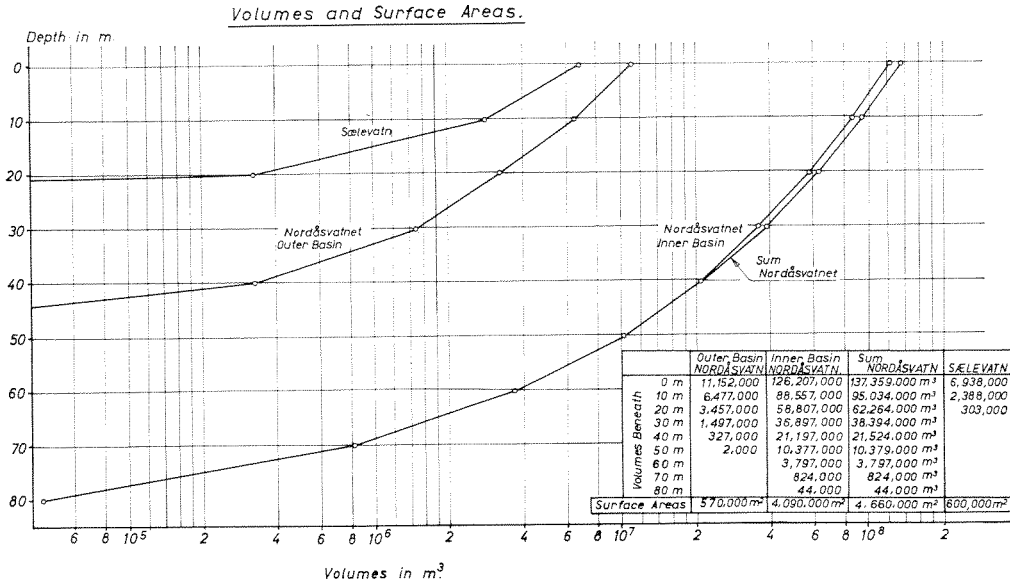


Fig. 4.

In wintertime Nordåsvatnet is often covered with ice, except at the outlet at Straume and across the ridge at Bønes where a high surface water velocity keeps the ice from forming.

Sælevatn, a shallow lake (maximum depth 26 metres) north of Straume also belongs to the system. It is connected to Nordåsvatnet through a horizontal shallow canal approximately 100 metres long. The interchange through this canal is small and the flow consists mainly of brackish runoff water. In volume, Sælevatn is approximately 5% of that of Nordåsvatnet, while the surface area is about 13% (Fig. 4).

The outlet from Nordåsvatnet or the canal at Straume consists of two canals each about 12 metres wide (Fig. 5). The tidal elevation in Knappesund regulates the direction and the velocity of the flow in the canal. The water level of Nordåsvatnet and its time dependency is thus a function of the tidal height outside Straume, the geometry and cross-sectional area of the canal, and the frictional resistance at Straume.

Tides outside Straume

Tidal elevations were measured inside and outside Straume 16 December 1960 and over a period of sixteen days in February/March 1961 (Fig. 12). Under the present conditions the canal at Straume has a choking effect of around 40% (Fig. 17), that is: the total tidal wave height inside Straume is approximately 40% of the total tidal wave height outside Straume.

Existing Canal at Straume.

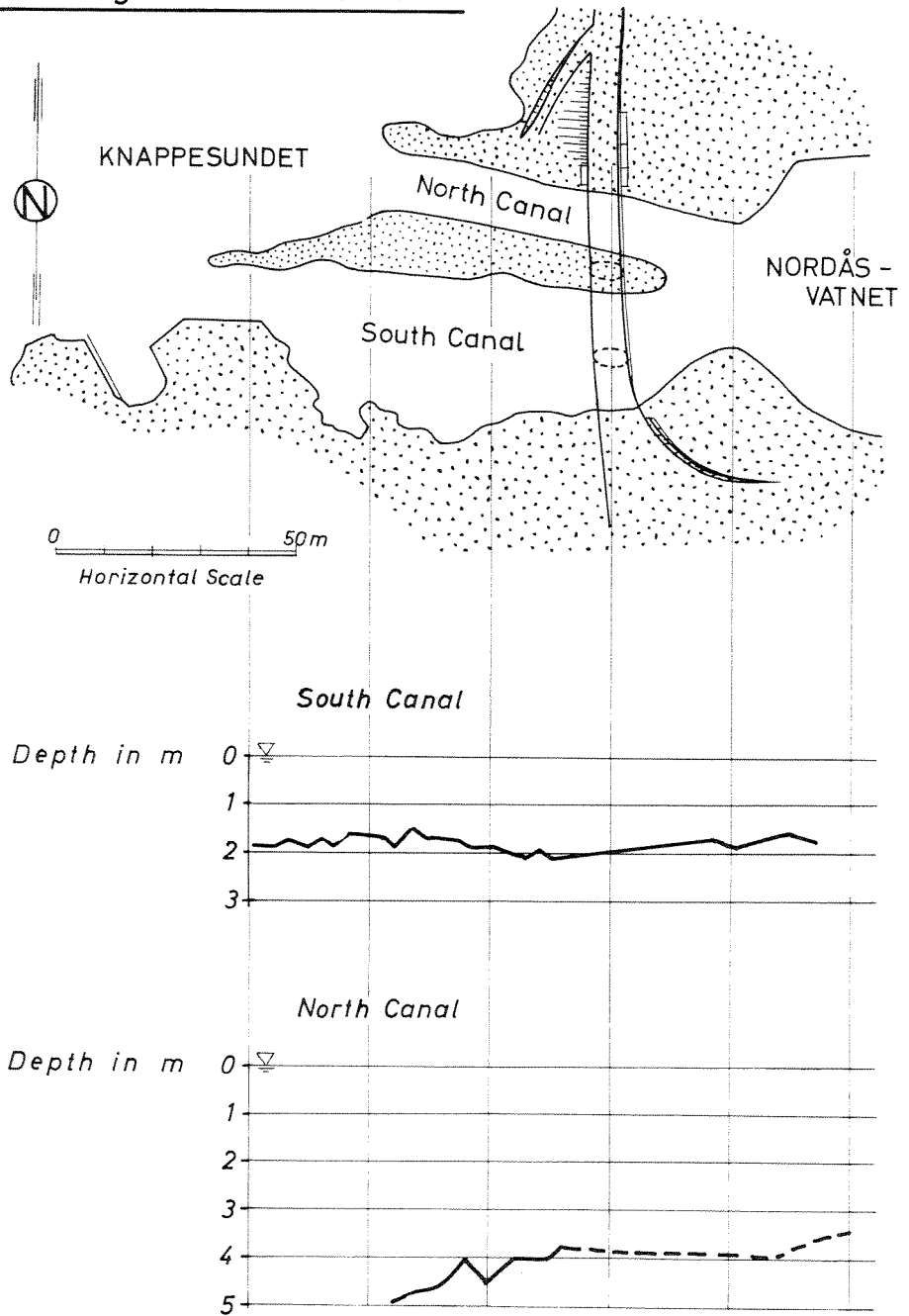


Fig. 5.

If the tidal elevations outside Straume are plotted as functions of time, the nearly sinusoidal relationship with the usual tidal period of 12.42 hours follows (Fig. 6).

$$h_u = H_u \cos \frac{2\pi t}{12.42} \quad (1)$$

where: h_u is the tidal elevation outside Straume as referred to the mean outside water level for that period;

H_u is the tidal amplitude outside Straume as referred to the mean outside water level for that period;

t is the time in hours ($t = 0$, when $h_u = H_u$).

This equation assumes that sills, ridges and bays in the Grimstadfjord have no influence on h_u , and that the inflow to and outflow from Nordåsvatnet have no feedback effect on h_u . The prior assumption appears to be allowable; however, the tidal periods of inflow and outflow in Knappesund are not quite equal. The time from low water to high water in Knappesund is on the average about 388 minutes while the time from high water to low water is 362 minutes. The very nature of this eccentricity implies that it is caused by the feedback of Nordåsvatnet into Knappesund.

For practical reasons it is convenient to express the tidal elevations outside Straume as a function of the tidal heights in Bergen. High and low water outside Straume precede those of Bergen by approximately 20 minutes. On account of

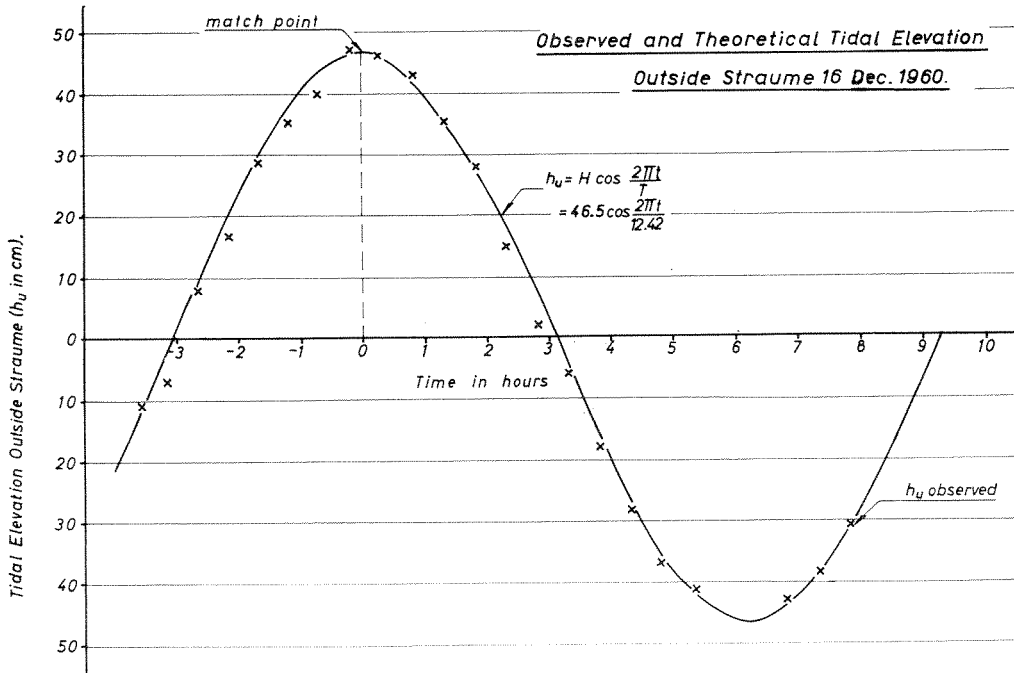


Fig. 6.

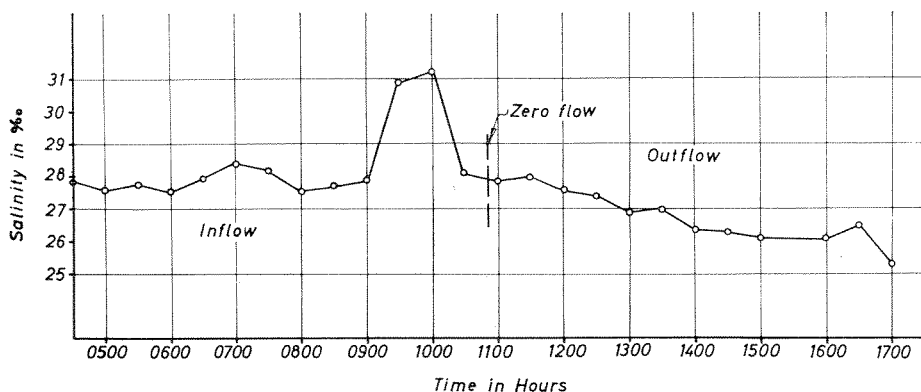
Salinity in South Canal 16 Dec. 1960.

Fig. 7.

the small eccentricity observed outside Straume the time difference between low waters is a few minutes longer and the high water time difference a few minutes shorter. The tidal amplitude outside Straume is approximately 84% of the one in Bergen as given by the Norwegian Tidal Tables (1961). Using these approximations the tidal elevations outside Straume can be expressed as a function of the tidal amplitude in Bergen and its time of occurrence.

$$h_u = 0,84 H_B \cos \frac{2 \pi}{12,42} (t + 1/3) \quad (2)$$

where: h_u is the tidal elevation outside Straume;

H_B is the tidal amplitude in Bergen;

t is time in hours ($t = 0$ when high water occurs in Bergen).

Equations (1) and (2) do not take into account effects of wind and barometric pressure. Small inaccuracies induced by neglecting these factors are overshadowed, however, by the practical importance of the equations.

The tidal problem inside Nordåsvatnet will be discussed in detail on pp. 57—60.

Salinities

In this preliminary investigation little attention has been paid to the measurement of oxygen, temperature and salinity.

Salinity was, however, measured in connection with the determination of velocity in the outer basin by using bacteria as tracers (Fig. 8). Salinity samples were also taken at the time of flow measurements in the canal at Straume (Fig. 7). During the inflow through the canal at Straume the salinity of the water

Salinity in Nordåsvatnet 14 Dec. 1960.

(Inflow through Canal at Straume started at 1530 Hours).

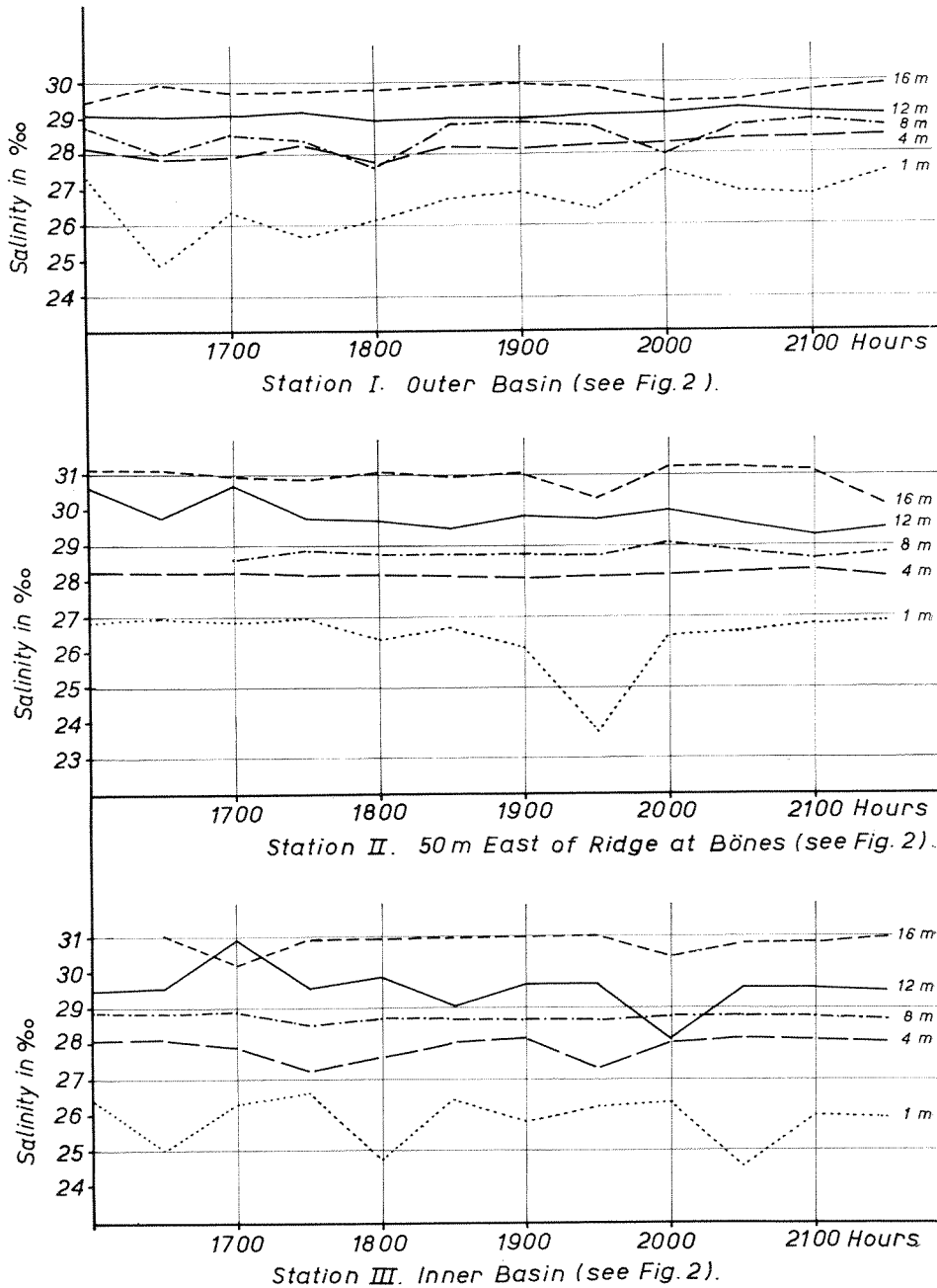


Fig. 8.

was about 28‰ except during the last two hours when it increased to 31‰. This implies that the water with the salinity of 28‰ was mostly water which was discharged from Nordåsvatnet during the previous cycle. An average salinity of 26–27‰ of the outflowing water seems to confirm this hypothesis of a certain pendulum effect. Thus, it is only during the last two hours of the inflow cycle that fjord water with high salinity flows into Nordåsvatnet. The only place within Nordåsvatnet itself where a salinity increase was slightly noticeable during the inflow cycle was at shallow depths in the outer basin. At other points in Nordåsvatnet the salinity seemed fairly constant during inflow at Straume. The salinity gradient with depth was, however, quite marked (Fig. 8). During the measurements Nordåsvatnet was covered with ice which may have limited the surface movements of the water. The observations seem to indicate a slightly more saline water in the inner basin than in the outer basin at the depth of 16 metres. The surface water in the inner basin, is, however, less saline than in the outer basin.

The bottom water of Sælevatn has a salinity of about 15‰ and a high content of hydrogen sulfide. A relative long period with cold and dry weather will cool and replace the regular brackish surface water in Sælevatn and expose the bottom water with its offensive H_2S odour.

The varying salt content and watersurface height of Nordåsvatnet make the technique of determining interchanges by measuring inflow and outflow salinity difficult to apply.

Freshwater inflow

Polluted freshwater enters Nordåsvatnet through sewers and natural streams (Fig. 11). Expressed as a percentage of the tidal interchange the freshwater inflow is small. However, due to its density and the location of the outlets, it is still important in the delicate balance of water quality in Nordåsvatnet. In the bay of Fjøsanger, the runoff forms, especially in summer, a dirty, unesthetically-looking surface layer. The discontinuity layer of density is also dependent upon the fresh-water inflow.

No attempt has been made to measure the freshwater inflow to Nordåsvatnet and it must therefore be computed from rainfall and evapo-transpiration data. With drainage areas of Nordåsvatnet and Sælevatn, located in a typical Norwegian west coast district, the orographic effect is very large. It is therefore not only difficult to find evapo-transpiration data, but also to find representative rainfall data. In Fig. 9 is plotted the average monthly precipitation in Fana as observed from 1896 to 1943. The numbers are taken from "Nedbøren i Norge" (1949) and are representative of the lower altitudes of the drainage areas. Also plotted is the comparative evapo-transpiration. The evapo-transpiration data are according to cand. real. UTAAKER at the University of Bergen, and are only approximate since no actual measurements have been carried out. The drainage

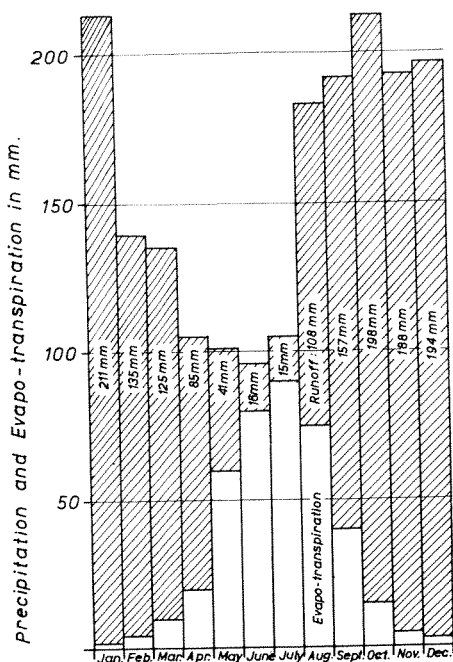
Precipitation and Evapo-transpiration for Fana.

Fig. 9.

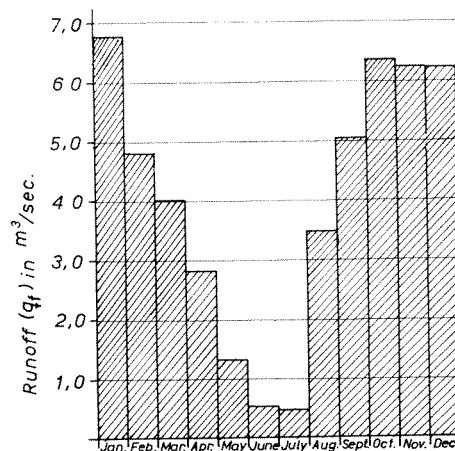
Freshwater inflow to Nordåsvatnet(Drainage area = 86 000 000 m²).

Fig. 10

areas (Fig. 1) compose c. 86.0 km² of which 74.3 km² empty directly into Nordåsvatnet and 11.7 km² into Sælevatn. The average monthly runoff can thus be computed and is plotted in Fig. 10.

On 16 December 1960 a total outflow of 2 220 000 m³ per tidal cycle was measured through the canal at Straume (Fig. 14). The average freshwater inflow for December is 6.23 m³/sec. For a tidal period of 12.42 hours the freshwater inflow is 278 000 m³ or about 12.5% of the total tidal outflow for the same tidal period. This percentage figure will of course vary greatly with the freshwater inflow and the tidal amplitudes. If the evaporation from the free surface of Nordåsvatnet was equal to the inflow from streams and sewers, the flow in the canal at Straume would describe a nearly odd symmetrical function (especially for small tidal amplitudes). However, the usual net freshwater inflow tends to lower the inflow at Straume and to increase the outflow. Although small, the freshwater inflow must nevertheless be included in the tidal interchange computations.

Sewer outlets

The number of sewer outlets in Nordåsvatnet has in the last 10–15 years shown a marked increase. No figures exist of the actual sewage flow or any part of it and one can only estimate the flow. The surrounding population numbers

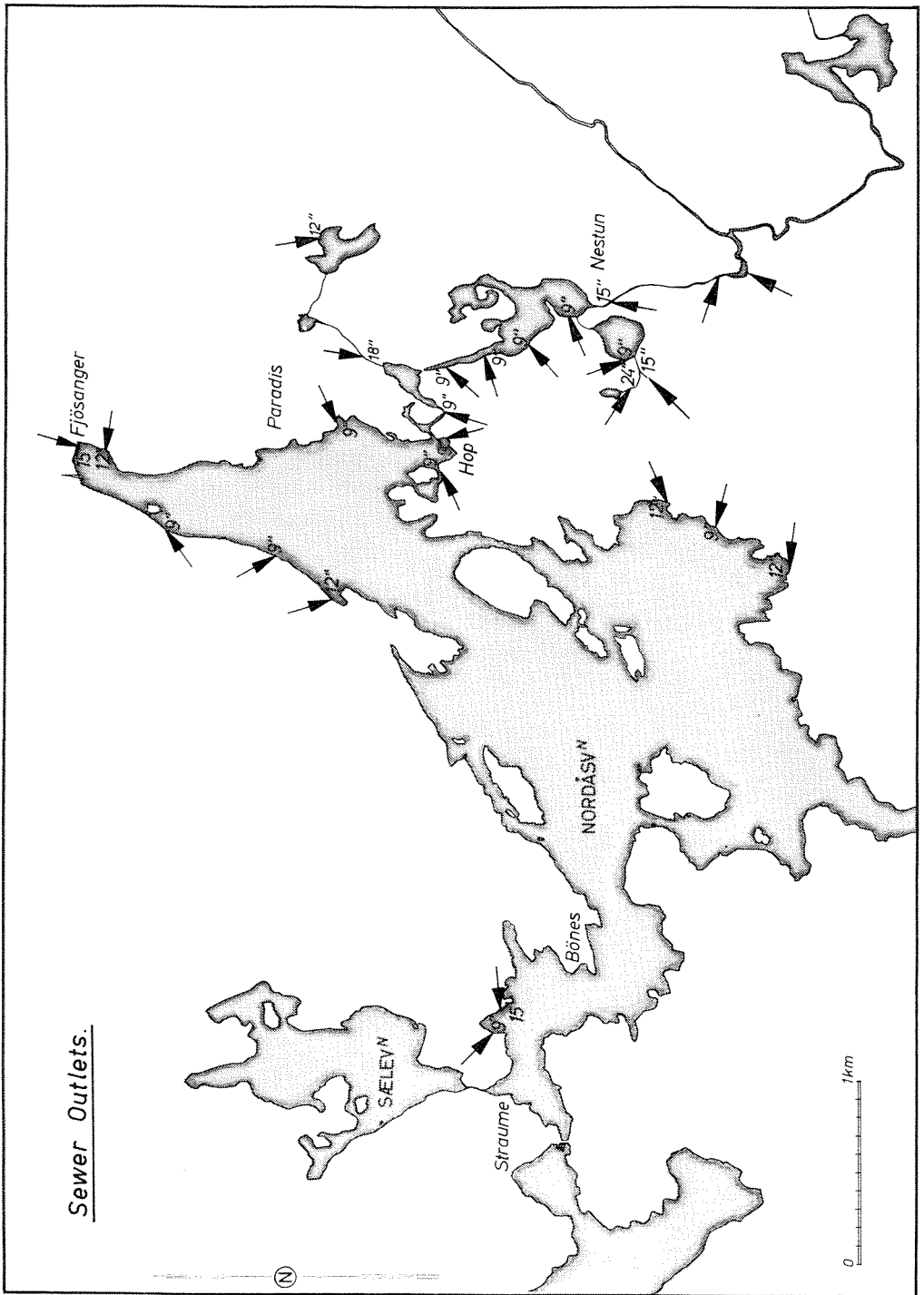
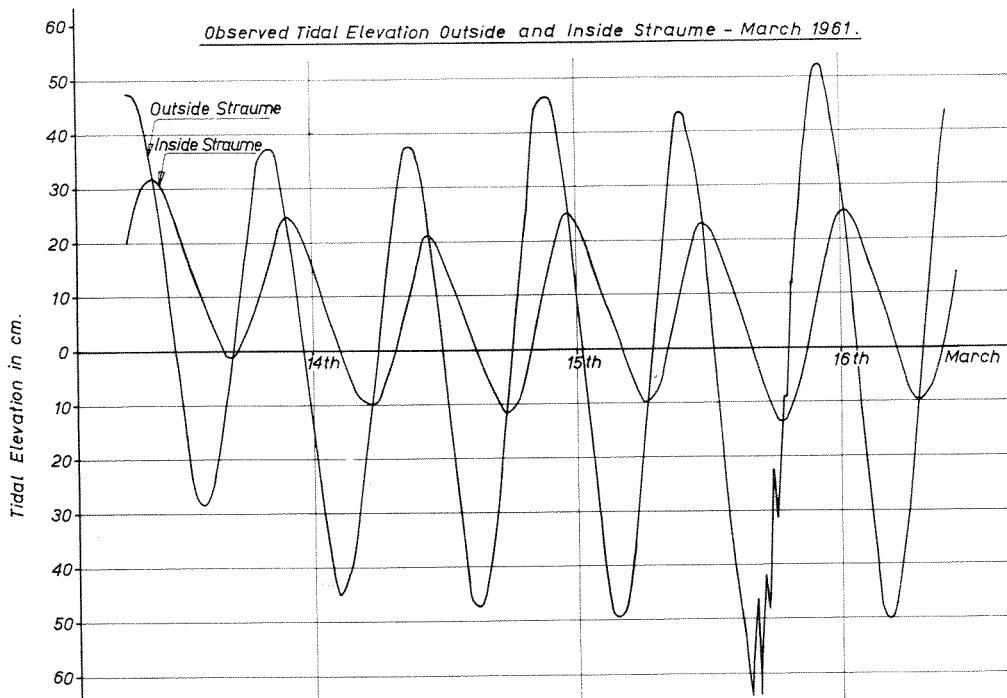


Fig. 11.



approximately 20 000, with a consumptive use of about 150 liters per day per capita. The result is a waste water flow of 3 million litres per day or about $0.035 \text{ m}^3/\text{sec}$. This estimate does not include the industrial waste water. In most cases the sewage has passed through septic tanks before entering the sewer system.

The amount of sewage discharging into Nordåsvatnet and its tributaries is fairly constant throughout the year. When expressed as a percentage of the freshwater inflow the sewage discharge is largest in the dry months of May, June and July.

Fig. 11 is a map of the most important sewer outlets. From this map it is evident how the surface water of the bay of Fjösanger and Paradis may become seriously polluted.

EXISTING INTERCHANGES

Elevations

Right from the beginning of this investigation it was obvious that the tidal heights inside Nordåsvatnet had to depend upon the tidal heights in the fjord outside the canal at Straume. It was also clear that the relationship between the tidal elevations outside and inside the canal at Straume depended upon the size, the geometry, and the frictional resistance of the canal itself. To observe this relationship in nature tidal elevations were measured simultaneously outside

and inside the canal at Straume (Fig. 12). In Nordåsvatnet itself no surface gradient could be measured. This simplifies the problem and means that the tidal elevations recorded in Nordåsvatnet near the canal at Straume, are representative of the whole part of the fjord situated inside the canal.

A time lag of about 110 minutes exists between high water outside and inside Straume. The same time lag for low waters is approximately 148 minutes. Translated into elevations this means that high water outside Straume is dampened or choked by the time it reaches Nordåsvatnet to approximately 56% of its original value. The same figure for the outside low tidal water is about 25%. The choking coefficient, defined as the total inside tidal wave height divided by the total outside tidal wave height is thus about 0.40. The time lags and choking coefficient will, as expected, vary somewhat with the outside tidal amplitude (Fig. 17). The shape of the inside tidal elevation curve (Fig. 12) is quite different from the one outside. However, with an increasing, choking coefficient the inside tidal curve would approach that of the nearly sinusoidal outside tidal curve. The positive displacement of the inside tidal curve from the zero of the outside tidal curve (Fig. 12) is caused by the freshwater inflows and the variation in canal area with tidal elevation at Straume.

The odd oscillations observed right after low water outside Straume 15 March 1961 (Fig. 12) are of unknown origin. The oscillations appear to have the form of a dampened shock wave with a period of about 20 minutes.

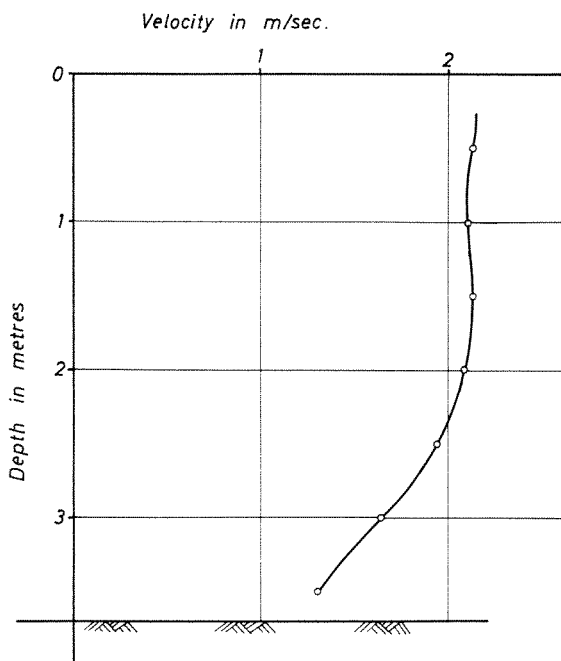


Fig. 13.

The influence of Sælevatn on the tidal elevations in Nordåsvatnet is difficult to define. The narrow canal into Sælevatn acts somewhat similar to the one at Straume, only its occurrence of inflow and outflow are more irregular. The effect of Sælevatn besides that of the freshwater inflow to Nordåsvatnet is, however, small and is included in the "effective area" of Nordåsvatnet.

To sum up the main features of the interchange of water one may say that the ridge at Bønes influences the quality of the deeper water in the inner basin while the canal at Straume regulates both the quality and the quantity of the water in Nordåsvatnet.

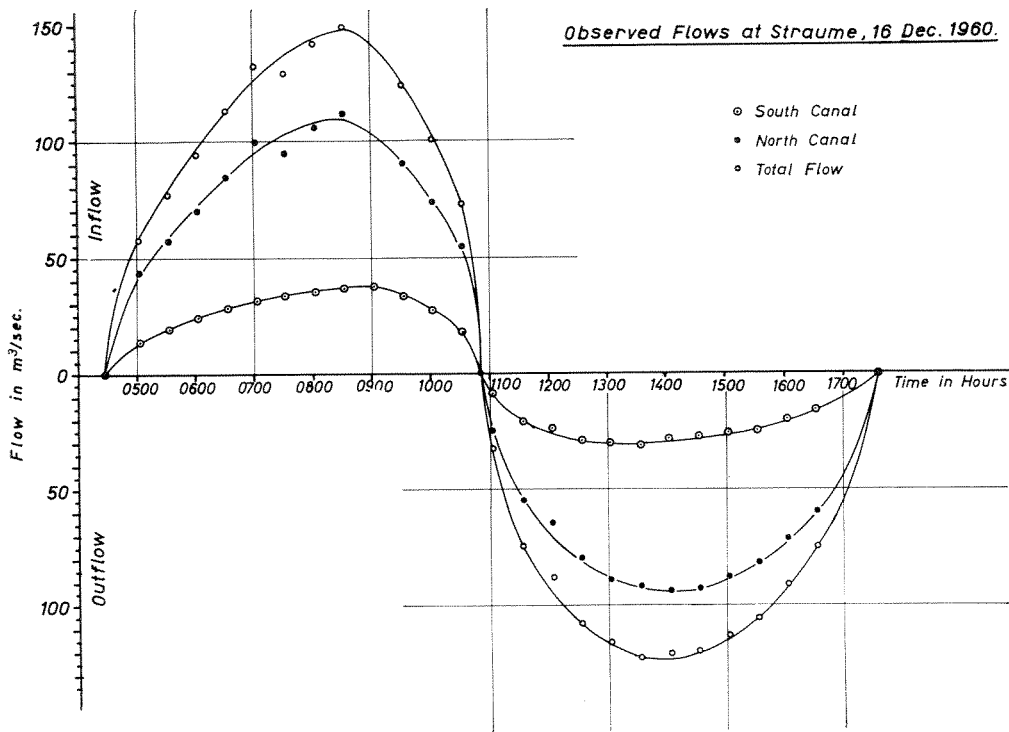


Fig. 14.

Velocities

The flow of water through the canal at Straume on 16 December 1960 was determined by stream gaging (Fig. 14). The velocity was measured at 6/10 of the depth, at 12 different sections, every half hour, using two Ott propeller current meters. A representative velocity profile is presented in Fig. 13. The maximum velocities were observed in the north canal where velocities as high as 264 cm/sec were measured during the inflow. The maximum average velocities computed for the whole cross-sectional canal area were 2.15 m/sec and 2.06 m/sec during inflow and outflow respectively. No special plot has been made of these data due to an irregular outside tidal amplitude.

Mass transfer

To compute the mass transfer through any section of Nordåsvatnet the change in tidal height (Fig. 12) is simply multiplied with the appropriate surface area and the freshwater inflow added or subtracted.

The total surface area of Nordåsvatnet and Sælevatn is approximately 5 260 000 m² (Fig. 4). Due to the choking effect of the small canal leading into Sælevatn an effective surface area of 5 080 000 m² is used in this report as the total area of Sælevatn and Nordåsvatnet. By applying figure 17 one can for a

known amplitude compute the mass transfer. Selecting an outside amplitude of 50 cm, a choking coefficient of 0.35 and, leaving out the freshwater inflow for the present, a volume of 1 780 000 m³ is found to pass through the canal at Straume. Approximately 80.5 ‰ of this or 1 430 000 m³ will flow across the ridge at Bønes. The influence of a net freshwater inflow is, of course, to lower the inflows and to raise the outflows.

The graph of discharge through the canal at Straume shown in Fig. 14 is not a representative one. The outside amplitude of 46 cm, as shown in Fig. 6, was not symmetrical and resulted in a larger inflow than outflow. The distribution of flow, with the north canal carrying approximately 75 ‰ of the total, should, however, be valid. The peaked inflow and the rounded outflow curves are also typical. This effect is caused by the variation in the cross-sectional area of the canal with the tidal heights and is more marked the larger the outside amplitude. The variation in the cross-sectional area with tidal height together with the freshwater inflow also results in a longer outflow than inflow period. This is because there is more water to go out than in, and because the flow resistance of the canal is larger during outflow than inflow due to the smaller cross-sectional area and a more disadvantageous geometry.

The outflow of sewage and polluted waters from Nordåsvatnet is dependent upon the tidal interchange of water in Nordåsvatnet. To increase the tidal interchange by increasing the flow in the canal at Straume is therefore a logical step in the direction of cleaning up Nordåsvatnet.

THEORETICAL CONSIDERATIONS

Mathematical formulation

The problem of tidal interchanges in Nordåsvatnet can be mathematically described by combining three main suppositions.

1. The change in elevation inside Nordåsvatnet is directly proportional to the time integral of the water added or subtracted by the canal at Straume and the freshwater inflow.
2. The flow in the canal at Straume is a function of the geometry, length and cross-sectional area of the canal, and the difference in outside and inside tidal elevations.
3. The tidal elevation outside Straume can be expressed as a function of time. This formulation ignores the effects of wind, temperature and variation in barometric pressure.

Expressed in mathematical terms:

The integration effect that Nordåsvatnet and Sælevatnet have upon the incoming water can be expressed in the following manner:

$$\frac{dh_i}{dt} = \frac{1}{A_N} (q + q_f) \quad (3)$$

where: q is the instantaneous flow rate in the canal;
 q_f is the freshwater inflow rate;
 h_i is the tidal elevation inside Straume;
 A_N is the effective surface area of Nordåsvatnet and Sælevatnet;
 t is time.

As previously mentioned, the effective surface area of Nordåsvatnet and Sælevatnet is here taken as 5 080 000 m² and the freshwater inflow is treated as a constant (Fig. 10).

The flow in the canal is described by Manning's empirical formula:

$$q = \frac{R^{2/3} A}{n} \sqrt{S} \quad (4)$$

where: q is the instantaneous flow rate in the canal;
 A is the cross-sectional area of the canal;
 R is the hydraulic radius of the canal, defined as the cross-sectional area of the canal divided by the wetted perimeter (Fig. 16);
 n is a coefficient of roughness;
 S is the slope of the energy line.

Manning's formula which is intended for uniform flow can also be written:

$$q = \frac{R^{2/3} A}{n} \sqrt{\frac{\Delta h}{L}} \quad (5)$$

where: L is the length of the canal;

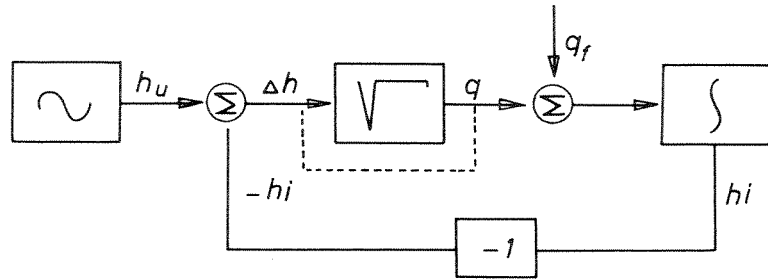
Δh is the difference in elevation of the water surface over the length L .

The uniform flow assumption embodied in equation (5) is not a stringent one, and the formula generally gives satisfactory results when applied to natural canals. In this study Δh is the difference in tidal elevations between outside and inside Straume ($\Delta h = h_u - h_i$) and L the total length of the canal. Since Δh has both positive and negative values we rewrite equation (5) in the form:

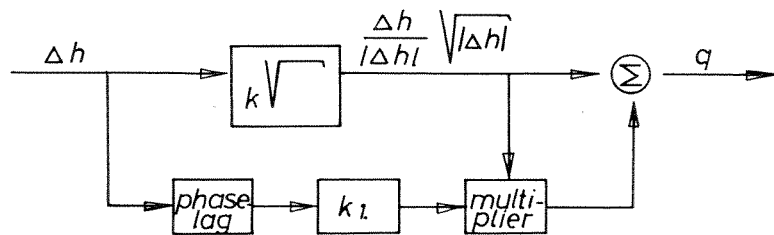
$$q = \frac{R^{2/3} A}{nL^{1/2}} \frac{\Delta h}{|\Delta h|} \sqrt{|\Delta h|} \quad (6)$$

The roughness coefficient used here is one of 0.043. This value was found through 48 different current measurements in the canal at Straume and ranged only from 0.036 to 0.047 proving the worth of Manning's formula. The value of 0.043 also agrees closely with the one given for rock channels by Chow (1959).

The total cross-sectional area of the canal is about 63.5 m² and average length of the canal approximately 100 metres. In this study neither the cross-sectional area of the canal nor the hydraulic radius can be looked upon as constants since both vary with tidal elevations.



a. Complete circuit.



b. Manning's formula (detailed).

Fig. 15.

As earlier stated, the tidal elevations outside Straume can be expressed as a sinusoidal function with a period of 12.42 hours, i.e.

$$h_u = H_u \cos \frac{2\pi t}{T} \quad (1)$$

where: T , the tidal period, is equal to 12.42 hours.

Substituting equations (1) and (5) into equation (3):

$$-\frac{q + q_t}{A_N} = \frac{2\pi H_u}{T} \sin \frac{2\pi t}{T} + n^2 L \frac{d}{dt} \left(\frac{q^2}{R^{4/3} A^2} \right) \quad (7)$$

setting $q = VA$:

$$-\left(\frac{AV}{A_N} + \frac{q_f}{A_N} \right) = \frac{2\pi H_u}{T} \sin \frac{2\pi t}{T} + n^2 L \frac{d}{dt} \left(\frac{V^2}{R^{4/3}} \right) \quad (8)$$

Instead of trying to solve equation (8) mathematically the system of equations, (1), (3) and (6), were fitted on an analog computer for solution of the problem.

Analog computer application

Fitting of the problem

A machine solution of the equations is obtained by using the signal flow diagram shown in Fig. 15a. The oscillator, to the left on the diagram, drives the model by generating h_u as an A.C. voltage. The difference in water level,

$\Delta h = h_u - h_i$, forces a flow, q , through the model. The freshwater inflow, q_f , is added and the time integral of the total flow taken to obtain h_i . h_i is fed back and subtracted from the outside tide, h_u , to give Δh .

The computer, containing only 10 amplifiers and one multiplier, limits to a certain extent the detailed computer programme. Proceeding from the left in Fig. 15a the computer programme contains the following:

1. Generator for the outside tidal height h_u .

This consists of two integrators in series, which continuously solve the differential equation:

$$h_u + \omega^2 \ddot{h}_u = 0 \quad (9)$$

whose solution is the desired function:

$$h_u = H_u \sin(\omega t + f) \quad (10)$$

The amplitude H_u and the phase angle f are here defined by the initial conditions (voltage across the capacitors) of the two integrators. Since only the steady state solution is of interest, f has no significance.

The angular frequency ω is chosen as low as 1/4 because of the relatively slow electromechanical components (multiplier and recorder). This gives a period, T , of 25.2 seconds in the model and a time scaling factor of $\frac{25.2}{44700} = 0.000564$.

The initial condition in the first integrator is set equal to H_0 , scaled as 1 volt pr. cm of tidal amplitude, and the second integrator starts with an uncharged capacitor, giving zero phase angle.

2. Computation of the flow q .

Manning's formula

$$q = \frac{R^{2/3} A}{n L^{1/2}} \frac{\Delta h}{\Delta h} \sqrt{|\Delta h|} \quad (6)$$

was evaluated with different degrees of accuracy:

a. The area and hydraulic radius assumed constant:

$$q = k \frac{\Delta h}{|\Delta h|} \sqrt{|\Delta h|} \quad (11)$$

where k is a constant.

This gives a first approximation to the values of q and h_i . The shape of the curves for q and h_i is rectangular and triangular respectively and symmetrical about the mean water level.

b. The time variation in A and R was taken into account by writing Manning's formula in the form:

$$q = k(1 + k_1 \Delta h) \frac{\Delta h}{|\Delta h|} \sqrt{|\Delta h|} \quad (12)$$

where k_1 is a new constant computed by assuming a certain H_i .

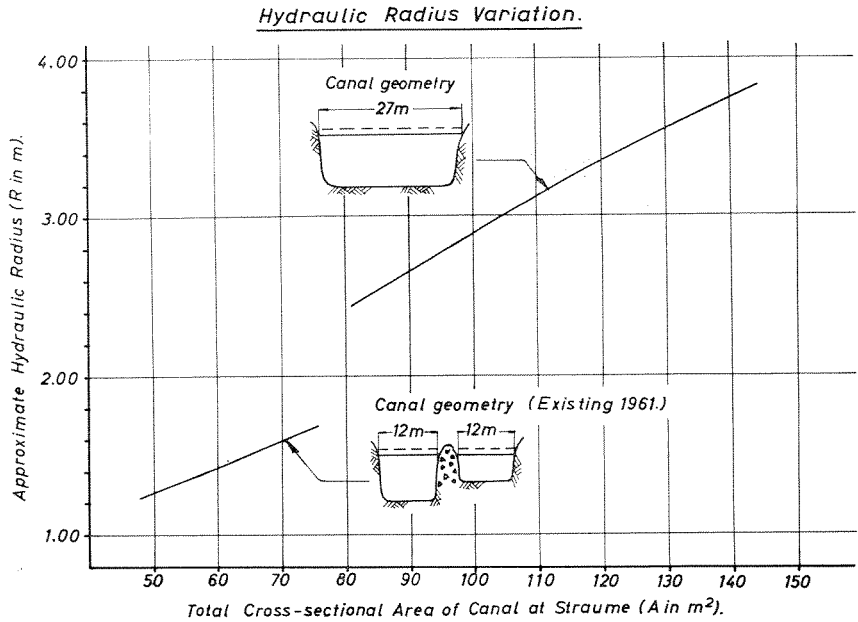


Fig. 16.

This approximation gives the characteristic non-symmetrical curves in q and h_i caused by the fact that inflow takes place through a larger cross-sectional area than the outflow. The channel thus has a rectifying effect, making the average inside tidal elevation higher than average outside tidal elevation even without freshwater inflow.

c. A more realistic approximation is obtained by giving $k_i \Delta h$ a certain phase shift with respect to Δh .

This phase shift must be computed on the basis of an estimation of the result. This improvement introduces a certain skewness in the curves caused by the fact that the extreme values of A and R occur somewhat later than h_{max} .

The final block diagram for the q computations is shown in Fig. 15b. The existence of only one multiplier is the major reason for this specific approach. The canal parameters are combined in the factors k and k_1 represented by variable resistors in the model. The scaling is 1 volt to 10 m^3/sec .

3. Freshwater inflow.

The freshwater inflow is represented by an adjustable D.C. voltage.

4. Computation of h_i .

The internal tidal elevation, referred to mean sea level, is related to the total flow by:

$$h_i = \frac{1}{A_N} \int_0^t (q + q_f) dt \quad (13)$$

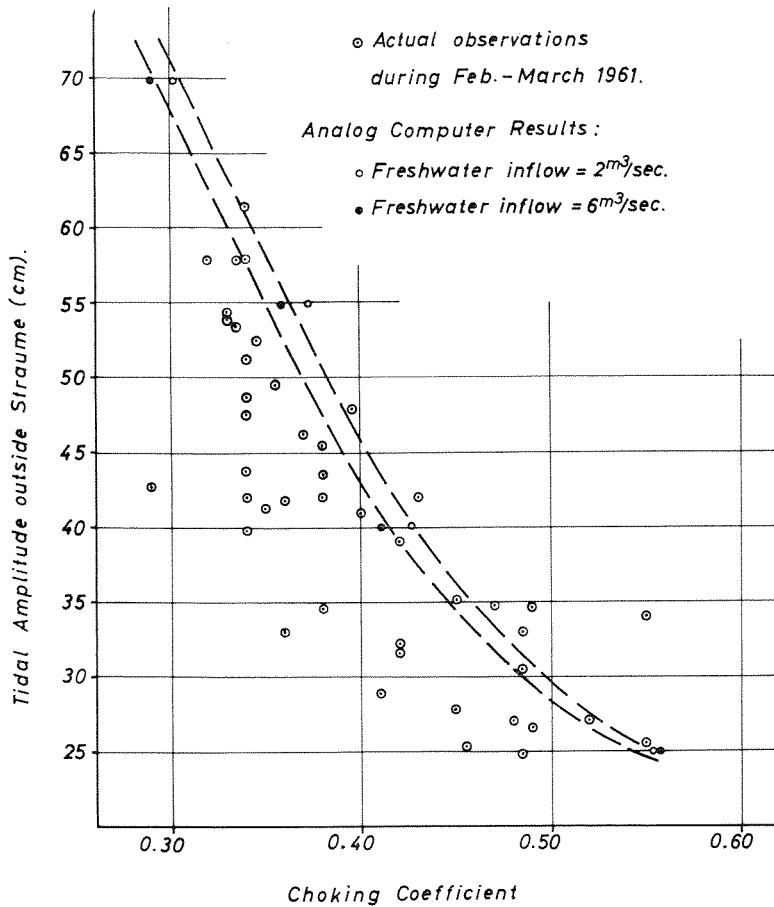
Choking Coef. vs. Tidal Amplitude.

Fig. 17.

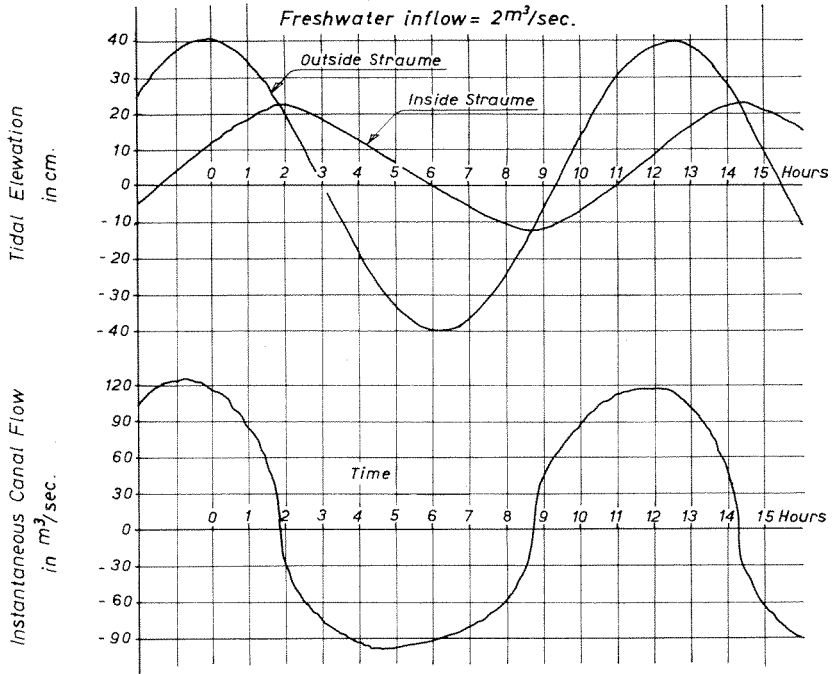
This is modelled by a single integrator. The area has to be multiplied by the time scaling factor, and a factor which gives h_i the proper scaling, namely 1 volt per cm.

Testing procedure

In this study only steady state solutions to the problem were considered. From Fig. 12 can be seen that in nature this is not always the case. To span the range of solution the outside tidal amplitude and the freshwater inflow were varied. For the existing cross-sectional area (63.5 m^2) four outside tidal amplitudes, namely 25, 40, 55 and 70 cm, and two freshwater inflows, 2 and $6\text{ m}^3/\text{sec}$ were used. These values were obtained simply by adjusting the voltages of h_u and q_f (Fig. 15a). From theory and existing interchanges values of k , k_1 and the phase lag could be computed. The value of A_N was the same for all runs.

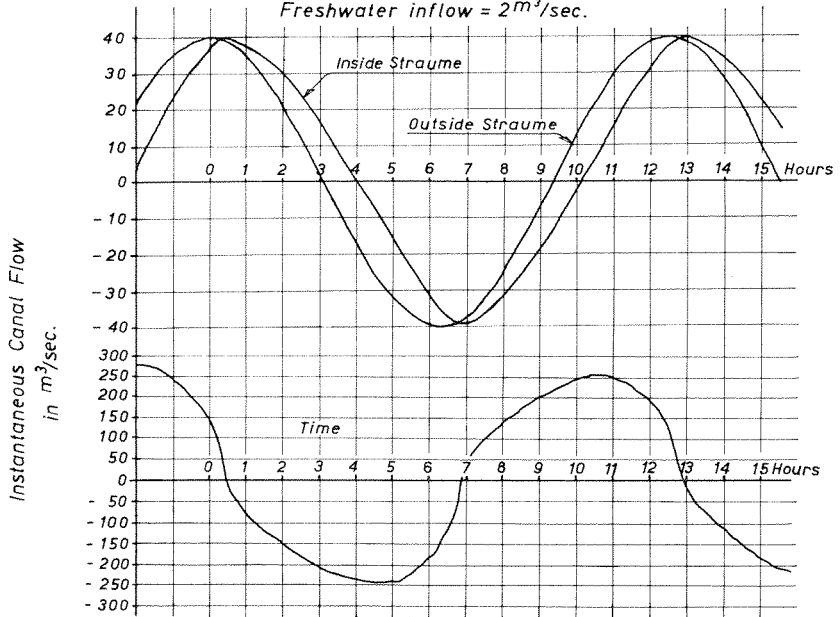
A-40-2.
(See table).

Canal Area = 63.5 m^2
Tidal Amplitude outside Straume = 40 cm
Freshwater inflow = $2 \text{ m}^3/\text{sec}$.



D-40-2.
(See table).

Canal Area = 135 m^2
Tidal Amplitude outside Straume = 40 cm.
Freshwater inflow = $2 \text{ m}^3/\text{sec}$.



Figs. 18 (above) and 19 (below).

In addition to the existing 63.5 m^2 , four theoretical cross-sectional areas were considered, 90, 115, 135 and 144 m^2 , and each run with two outside amplitudes, 80 and 110 cm with a freshwater inflow of $2 \text{ m}^3/\text{sec}$. The values of k , k_1 and the phase lag had to be computed from theory for each of the cross-sectional areas. With the parameters determined the computer was put to work with the values of h_u , h_i and q being continuously recorded (Figs. 18 and 19).

Results

Figs. 17 through 24 contain the data and results and are fairly self-explanatory. The most important data are summarized and tabulated in Table 1. A short description of each of the figures follows:

In Fig. 17 a comparison is made of the observed and theoretical choking coefficients. The spread in the observed choking coefficients is caused by non-symmetry in the outside tidal amplitudes and natural parameters such as wind, freshwater inflow etc. The computer results clearly indicate the inverse proportionality between outside tidal amplitude and choking coefficient. The choking coefficient decreases on the average about 3% by increasing the freshwater inflow from $2 \text{ m}^3/\text{sec}$ to $6 \text{ m}^3/\text{sec}$. Using the theoretically computed value of k in Manning's formula originally gave theoretical choking coefficients with about 6% higher values than shown in Fig. 17. Since many uncertainties were involved in computing k the theoretical value of k was lowered 12% to obtain the verification of the observed coefficients as shown in Fig. 17.

In Figs. 18 and 19 are plotted the raw data as produced by the recorder linked to the computer. For the sake of simplicity only two of the sixteen complete runs are shown. Time lags, periods of outflow and inflow, elevations, choking coefficients etc. can all be taken from Figs. 18 and 19 and the other similar plots. The shape of the inside tidal elevation curves and the flow curves can also be clearly seen from Figs. 18 and 19. The inside tidal elevation curve obviously approaches the sinusoidal outside tidal curve as the cross-sectional area of the canal increases. The peaked and rounded tendencies of the flow curves are also easily detected. The changes in the inside tidal elevations and the flow caused by increasing the freshwater inflow from $2 \text{ m}^3/\text{sec}$ to $6 \text{ m}^3/\text{sec}$ can be seen in Table 1.

In Figs. 20, 21 and 22 are plotted the maximum values of tidal elevation, velocity and flow as a function of the canal area. The lines drawn through the points with canal area of 90, 115, 135 and 144 m^2 do not necessarily pass through the points for the existing canal area since the canal geometry has been assumed differently (Fig. 16). From Fig. 20 it can be seen that if the canal area is increased from 63.5 m^2 to 135 m^2 the average high water elevation will rise about 23 cm. The increase in canal area would more than double the maximum flow rates in the canal (Fig. 22). However, the maximum velocities in the canal would stay about the same as under the existing conditions (Fig. 21). At a canal area

Maximum Tidal Elevation vs. Canal Area.

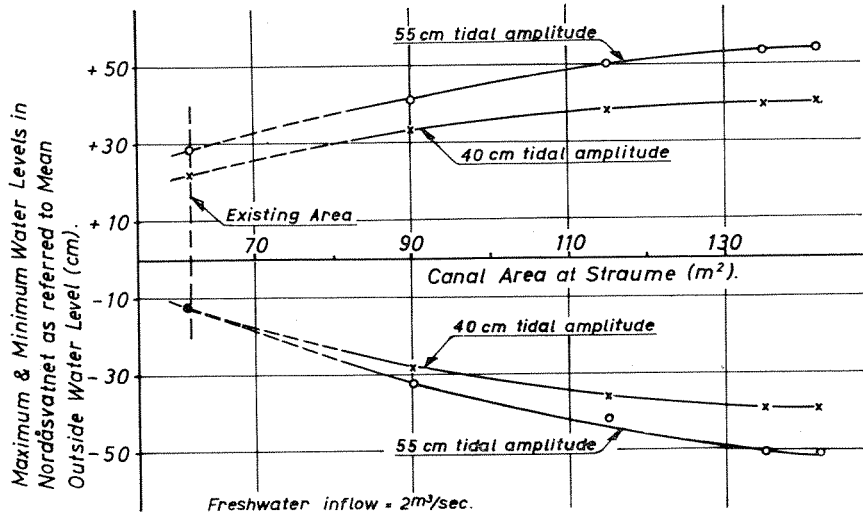


Fig. 20.

Maximum Canal Velocity vs. Canal Area.

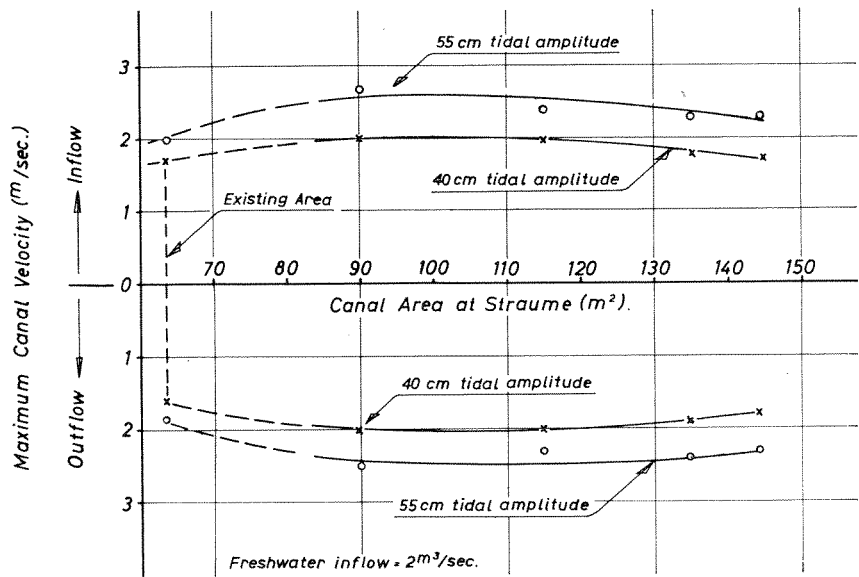


Fig. 21.

of approximately 100 m² the product of Δh and $R^{2/3}$ is maximum, resulting in the largest velocities with the assumed canal geometry.

For better to compare the time lags and periods, the instantaneous canal flow has been plotted versus the tidal period in Fig. 23. The similarity of the three curves with the larger canal area is due to the similar canal geometry.

The last figure (Fig. 24) with the theoretical choking coefficient plotted against the canal area probably gives the best overall view of the problem and its solution. The interchange of water in Nordåsvatnet is therefore also doubled. By increasing the canal area at Straume from 63.5 m² to 135 m² the average choking coefficient is about 0.97. This means that the present interchange of water in Nordåsvatnet would increase by a factor of approximately 2.4, and that the outside tidal pump was used to interchange the water of Nordåsvatnet with an efficiency of about 97%.

It is here in its place to point out that the computations regarding the interchanges are only quantitatively. No attempts have been made to define the qualities of the water that would be interchanged with a larger canal area.

Some non-consistant variations are observed in the values produced by the recorder. These errors are caused mainly by the "square-root-operator" and the balancing of the computer. Only in a couple of runs does this inaccuracy effect the curves produced. On the whole the accuracy of the results can be considered fully acceptable.

SUMMARY AND CONCLUSIONS

From a re-evaluation of the hydrographical conditions in Nordåsvatnet a theory was stated regarding the tidal interchanges in the fjord. The interchanges were quantitatively described through three equations:

1. The tidal elevations inside Straume were given by:

$$qdt + q_f dt = A_N dh_i \quad (3)$$

2. The flow in the canal at Straume was expressed by Manning's equation:

$$q = \frac{R^{2/3} A}{n L^{1/2}} \frac{\Delta h}{|\Delta h|} \sqrt{|\Delta h|} \quad (6)$$

3. The tidal elevations outside Straume were given by:

$$h_u = H_u \cos \frac{2\pi t}{T} \quad (1)$$

These three equations were fitted on an analog computer and the interchanges solved for the existing canal area at Straume and for four larger canal areas.

The theoretical results for the existing conditions were verified with actual observations.

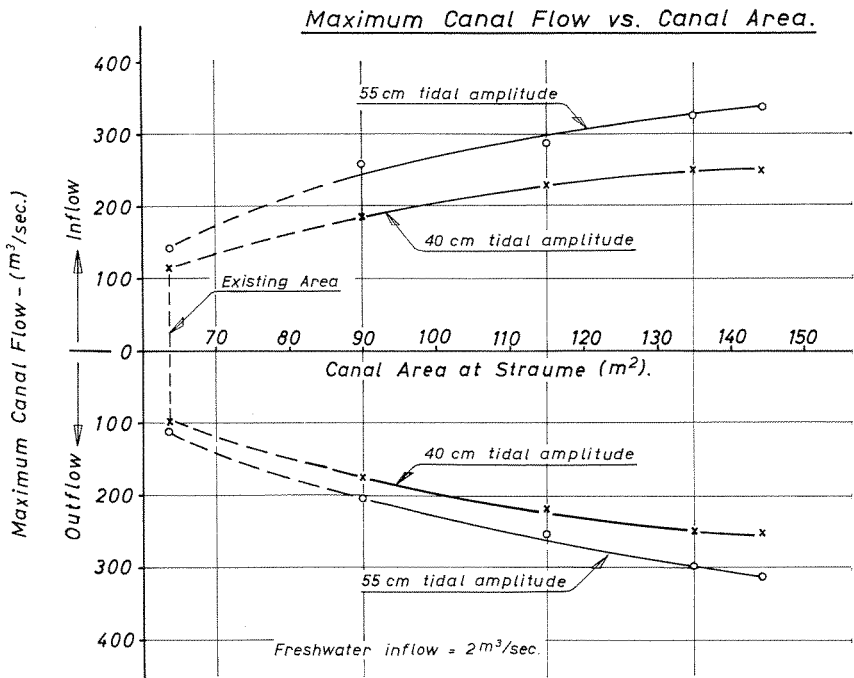


Fig. 22.

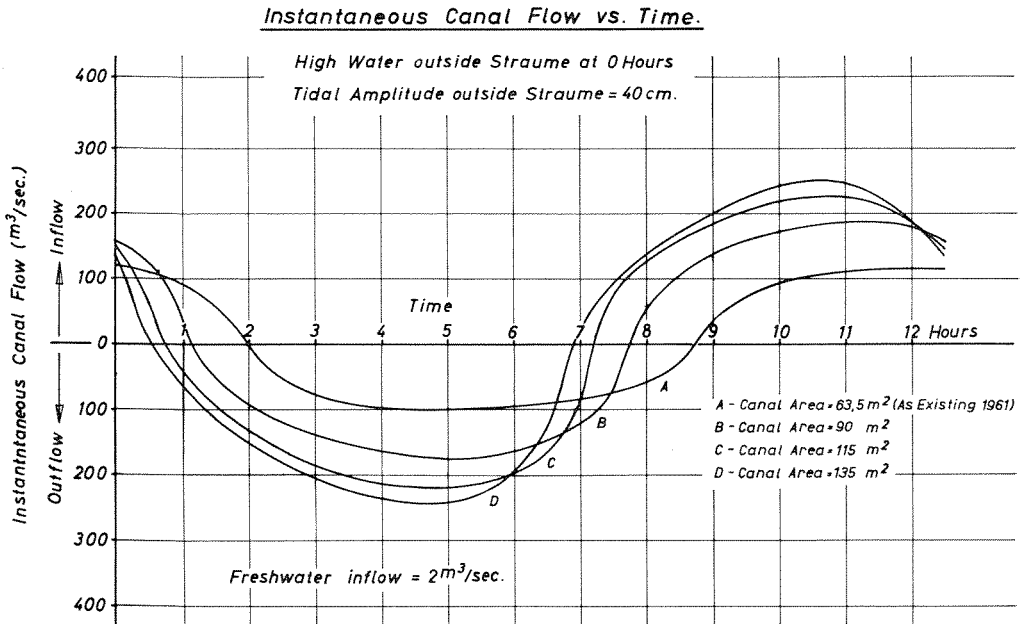


Fig. 23.

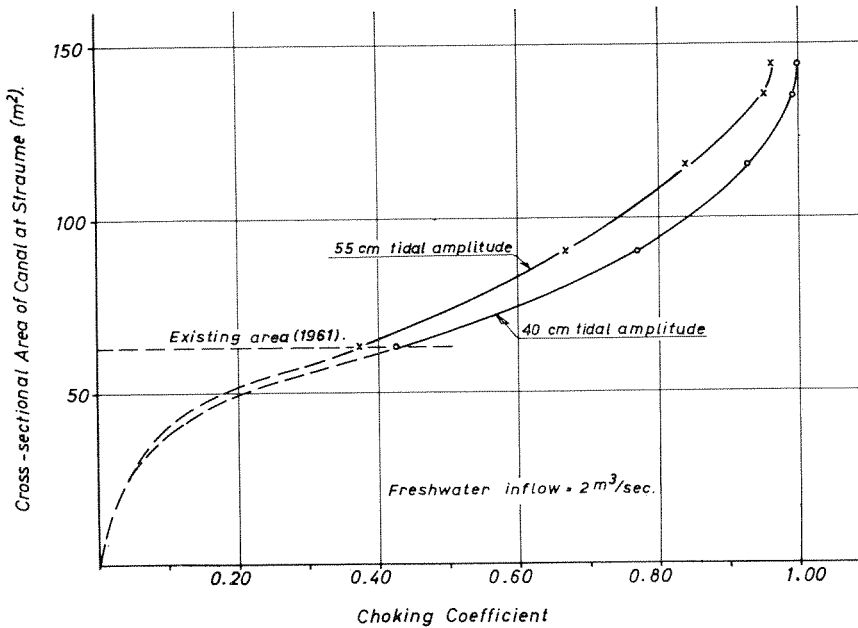
Choking Coef. vs. Canal Area.

Fig. 24.

The choking coefficient, defined as the total inside tidal wave height divided by the total outside tidal wave height, was found to be approximately 0.40 with the existing canal area at Straume of about 63.5 m². By increasing the canal area at Straume to 135 m² the choking coefficient reached 0.97. No change in the velocity of the water in the canal could be pointed out with this increase in the canal area.

Curves showing the instantaneous flow in the canal at Straume during the tidal cycle were plotted for four canal areas.

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Table 1. Theoretical analog computer results.

Area, amplitude and freshwater inflow	Maximum height inside Straume above MSL (cm)	Minimum height inside Straume above MSL (cm)	Choking coefficient	Maximum inflow through Straume canal (m ³ /sec)	Maximum outflow through Straume canal (m ³ /sec)	Maximum ingoing velocity through Straume canal (m/sec)	Maximum outgoing velocity through Straume canal (m/sec)	Duration of inflow through canal (hours)	Duration of outflow through canal (hours)	High water lag (hours)	Low water lag (hours)
A-25-2	16.0	-11.7	.555	87.5	81.0	1.3	1.3	5.8	6.6	1.8	2.3
-6	16.8	-11.0	.556	82.5	84.5	1.2	1.4	5.6	6.8	1.7	2.4
A-40-2	22.0	-12.2	.427	117	99.0	1.7	1.6	5.6	6.8	2.1	2.6
-6	22.8	-10.1	.411	111	101	1.6	1.7	5.4	7.0	2.0	2.6
A-55-2	28.9	-12.1	.373	143	108	2.0	1.8	5.5	6.9	2.1	2.7
-6	31.7	-7.8	.359	138	108	1.9	1.8	5.2	7.2	1.9	2.9
A-70-2	41.5	-0.8	.302	158	108	2.2	1.7	5.0	7.4	1.9	3.1
-6	44.0	3.4	.290	151	108	2.0	1.7	4.8	7.6	1.9	3.2
B-40-2	33.3	-28.3	.770	188	174	2.0	2.0	5.8	6.6	1.1	1.6
B-55-2	41.1	-32.3	.667	260	202	2.7	2.5	5.7	6.7	1.4	1.9
C-40-2	38.0	-36.1	.926	232	217	2.0	2.0	5.9	6.5	0.6	1.0
C-55-2	50.1	-42.0	.837	288	250	2.4	2.3	5.6	6.8	0.9	1.4
D-40-2	39.9	-39.4	.992	252	250	1.8	1.9	6.0	6.4	0.4	0.6
D-55-2	53.5	-51.0	.951	326	298	2.3	2.4	5.7	6.7	0.5	0.9
P-40-2	40.0	-39.8	.998	252	251	1.7	1.8	6.1	6.3	0.3	0.4
P-55-2	54.6	-51.3	.962	338	312	2.3	2.3	5.8	6.6	0.4	0.7

A - Canal area = 63.5 m² (existing area 1961)

B - ->- = 90 m²

C - ->- = 115 m²

D - Canal area = 135 m²

P - ->- = 144 m²

REFERENCES

- BRAARUD, T. and B. HOPE, 1952. The annual phytoplankton cycle of a landlocked fjord near Bergen. *Fiskeridir. Skr. Havunders.* 9 (16).
- CALDWELL, I. M., 1955. Tidal currents at inlets in the United States. *Proc. A.S.C.E.* 81 (716).
- CHOW, V. T., 1959. *Open-channel hydraulics*. Mc. Graw-Hill Book Co. N. Y.
- DET NORSKE METEOROLOGISKE INSTITUTT, 1949. *Nedbøren i Norge 1895-1943*. H. Aschehoug, Oslo.
- DRONKERS, J. J. and J. C. SCHÖNFELD, 1955. Tidal computations in shallow water. *Proc. A.S.C.E.* 81 (714).

- EINSTEIN, H. A. and R. A. FUCHS, 1955. Computation of tides and tidal currents — United States practice. *Ibid.* 81 (715).
- GAARDER, T., 1916. De vestlandske fjordes hydrografi. *Bergens Mus. Aarb.* 1915—1916.
- 1919. Die Hydroxylzahl des Meerwassers. *Ibid.* 1916—1917.
- KNUDSEN, M., 1956. *Hydrographical tables*. Tutein og Koch. Cph.
- KULLENBERG, B., 1946. Hydrografiska undersökningar i Stockholms skärgård under år 1945. *Bilaga till berättelse över Gatukontorets verksamhet* år 1945. Sth.
- 1949. Hydrografiska undersökningar i Stockholms skärgård under år 1946. *Tekniska meddelanden från Stockholms gatukontor* (1). Sth.
- undated report, c. 1955. *Vattenomsättningen i Stockholms inre skärgård och följderna av Oxdjupets upprensning*. Oceanografiska Institutet, Göteborg.
- MOHRMANN, J. C. J. and J. KESSLER, 1959. Waterdeficiencies in European agriculture. A climatological survey. *Intern. Inst. for Land Reclamation and Improvement*. Wageningen.
- MOSBY, H. and T. M. VOGELSANG, 1949. *Sjøbadene i Bergen*. A.S John Griegs Boktrykkeri, Bergen.
- NORGES SJÖKARTVERK, 1961. *Tidevanntabeller*. Stavanger.
- OTTER, J. R. H. and A. S. DAY, 1960. Tidal flow computations. *Engineer, Lond.*
- PETTERSON, H., 1946. Utlåtande angående föreslagen åtgärd för förbättring av vattenomsättningen i Stockholms hamnområde. *Bilaga till berättelse över Gatukontorets verksamhet* år 1945. Sth.
- ROSE, D., 1961. Der Einsatz elektronischer Grossrechneautomaten bei der numerischen Ermittlung der Gezeiten und Gezeitenströme. *Mathematik — Technik — Wirtschaft* (1).
- RUGE, H., 1961. Analogiregnemaskinen Mons. *Sentralinstituttet for Industriell Forskning*. SE 1511. Blindern, Oslo.
- STOKER, J. J., 1957. *Water Waves*. Interscience Publishers Inc., N. Y.
- STRÖM, K. MÜNSTER, 1936. Land-locked waters. *Skr. norske VidenskAkad. I. Mat.-Naturv. klasse* 1936 (7).
- SVERDRUP, H. U., M. W. JOHNSON and R. H. FLEMING, 1942. *The Oceans*. Prentice — Hall Inc. New Jersey.
- TAMBS-LYCHE, H., 1957. Daily observations of surface temperature and salinity at the Biological Station during 1951 and 1952. *Univ. Bergen Arb. 1957, naturv. R.* (7).
- VINDENES, B., 1944. On the tidal waters in the Nordåsvatn. *Fiskeridir. Skr. Havunders.* 7 (8).
- WIBORG, K. F., 1944. The production of zooplankton in a land-locked fjord. *Ibid.* 7 (7).

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