NORWEGIAN INSTITUTE FOR WATER RESEARCH

Blindern.

DETERMINATION OF FLOW BEHAVIOUR AND

FRICTION FACTORS OF SEA-WATER

MIXED WITH CaSi-DUST.

O-5/65.

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CONTENTS.

TABLES AND FIGURES.

I INTRODUCTION.

The disposal possibilities of a large amount of dust from FeSi, Si-metal
and CaSi productions as a result of melting processes by Fiskaa Verk in
Kristiansand, with the aim of sea water was reported previously. [1] During
the above mentioned work all study were based on the dust from FeSi pro-
duction, and it was thought that obtained results would be valid also for
other dust qualities in question. Later it was discovered by Fiskaa Verk
that dust from CaSi production had different characteristics than the other
categories, the mixture of which sea-water might result in different
flow properties.

The proposed transportation line for the disposal of dust is considerably
long, and, for economic reasons, the accurate determination of friction
factor was necessary. Therefore the flow behaviour of sea-water mixed with
varying concentrations of CaSi-dust was investigated through the rotational
viscometer. These data are essential for determining the flow parameters
necessary for the evaluation of friction factor. As a result of this
investigation the flow shows a combination of Bingham plastic and pseudo-
plastic behaviour at high dust concentrations, but only Bingham plastic
behaviour at lower concentrations.

The specific weight of CaSi-dust was measured by our institute with the
aim of pycnometer where carbontetrachloride was used as repressingwater.
Viscometric measurements were undertaken by the Central Institute for
Industrial Research, given range of which was not wide enough for the
accurate determination of flow parameters and thixotropy if it existed.
The friction factors obtained for both Bingham plastic and pseudo-plastic
flow behaviours will result in considerable differences in their correspon-
ding head loss values (see Tables 3 and 4).

II DISCHARGE CONDITIONS.

Previous experiments showed that to achieve effective density currents and
harmless discharge conditions at the discharge area, the specific weight of

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1 - 0-5/65 "En vurdering av transport og utslipp av oppslemmet SiO_2-støv
    fra Fiskaa Verk, Kristiansand, på stort dyp i sjøen", Norwegian Institute
    for Water Research, Oslo, September 1965.

discharged liquid should range between 1.074 and 1.132, corresponding concentrations for which were n = 200 g/l (1/5) and 100 g/l (1/10) respectively. The measured specific weights of the later dust from Si-metal and CaSi productions are 1.748 and 1.795 respectively, which are considerably different from the specific weight of FeSi.

As it is obvious from Fig. 1 and values given in Table 1, CaSi-dust concentrations are not the same for the corresponding suggested specific weights and remain within the range of n = 250 g/l (1/4) - 111 g/l (1/9). In relation with the achievement of a suitable density current, given flow velocities in the transportation system were about 0.80 - 1.50 m/sec. for 100 - 150 mm Ø pipes.

III   BEHAVIOUR OF SEA-WATER MIXED WITH CaSi-DUST.

The shear stress - shear rate curves (fluid flow curves)   representing the behaviours of very well known fluids under laminar flow conditions are given in Fig. 2 [2].

For comparision and because relationship between shear stress and shear rate can be denoted best on an arithmetic paper, the related viscometric data, as measured by the Central Institute for Industrial Research, are given in Figs. 3 and 4 as an arithmetic plot. Therefore in the following it is tried to estimate the behaviour of the liquid in question by making comparision between the curves given in Fig. 2 and Figs. 3 and 4. As it is obvious from the Figs. 3 and 4, it seems that in all cases flow initiated until a yield value of shear stress was overcome. The rest of the curves are approximately straight lines which can be assumed as increasing linearly with the stress as it is for Bingham plastic fluids [2] [3]. The common expression for this behaviour is given by the following relationship.

$$\tau - \tau_y = \eta \left(\frac{du}{dy}\right) \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (1)$$

where

2 - Behn, V.C., Sludge Flow Equations, Journal VPCF, July 1960, p. 731.

3 - Rich, L.G., Unit Operations of Sanitary Engineering, John Wiley and
    Sons Inc., 1961, p. 5-7.

τ = shear stress

τ_y = yield stress

η = coefficient of rigidity (slope)

$(\frac{du}{dy})$ = shear rate

The above given expression differs from Newtonian fluids only in that the relationship between, τ, and, $(\frac{du}{dy})$, does not go through the origin, as is found for the liquid in question.

On the other hand, as is experienced very often, a great deal of pseudo-plastics will plot straight lines on log-log paper. Our observations plotted on log-log paper partly introduce straight lines (see Figs. 5-9). In this case the relationship between shear stress and shear rate is given by the following power function.

$$\tau = k \ (\frac{du}{dy})^m \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (2)$$

where

m = flow behaviour index

k = flow consistancy index

Above information indicates very well that the behaviour of this liquid is non-Newtonian and involves more than one parameter. The accurate determination of these parameters for pipe line design is of great importance.

IV PRACTICAL DETERMINATION OF FLOW PARAMETERS AND FRICTION FACTORS.

The logaritmic plot of viscometric data for dust concentrations higher than 125 g/l, demonstrate the combination of Bingham plastic and pseudo-plastic behaviours. The arithmetic plot of the same data gives, however, impression that the possibility of Bingham plasticity may exist for the whole series of data, and that pseudo-plasticity might be due to non-uniform distribution and settling of dust particles at low flow rates during the operation of viscometer. It is believed that the properly stirring of the liquid before operation might eliminate or at least mini-mize the assumed influence of non-uniform distribution and settling of particles.

The calculated values of behaviour index $(m < 1)$ [4] showed that the liquid in question might show whether Bingham plastic or pseudo-plastic behaviours. For the solution of our problem the consideration of each case seperately was thought to be necessary.

1. Bingham plastic.

For the calculation of friction factor Reynolds number, Re, for Bingham plastic behaviour is expressed by Eq. 3.

$$Re = \frac{UD\gamma}{\eta} \quad \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \quad (3)$$

where

        U = mean velocity of flow in pipe
        D = inside diameter of pipe
        γ = specific weight
        η = coefficient of rigidity

The data needed for the calculation of Reynolds number such as mean velocity of flow in pipe and inside diameter of pipe were determined as a result of previous experiments, required coefficients of rigidity were obtained from the arithmetic plot of viscometric data (Figs. 3 and 4). Sea-water containing varying dust concentrations and corresponding coefficients of rigidity are given in Table 2. Friction factors (λ) for determined flow velocities and pipe diameters are given in Table 3. Friction factors were obtained from Moody diagram through the curve for smooth pipes (Fig. 10).

2. Pseudo-plastic.

Reynolds number for pseudo-plastic flow is expressed by the following equation (see ref. 2, p. 736).

$$Re = \frac{8U^{2-m} D^{m} \gamma}{k (\frac{6m + 2}{m})^{m}} \quad \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \quad (4)$$

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4 - Perry, J.H., Chemical Engineering Handbook, Mc Graw-Hill Book Comp. Inc., 1963, p. 5-36.

In the above equation it is necessary to determine the flow consistancy index, k', and flow behaviour index, m'. Referring to the curves given on log-log papers, m' can be read as the slope of the curves, and k' the intercept on the, τ - axis, for $(\frac{du}{dy}) = 1$.

As pointed out by Behn [5] the rotational viscometer equations utilized are not based on any particular shear stress – shear rated model and can not be used directly to evaluate the flow parameters. Therefore the interchange of rotational viscometer data to pipe line data is necessary. For the interchange in question the following equation was used.

$$k' = k \; (\frac{3\,m' + 1}{4m'})^m \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

where

      k = flow consistancy index of rotational viscometer data
      m = flow behaviour index of   "   "   "

The determination of, m', through the existing data was not possible, so, k', was evaluated by assuming, m, to be very close in value to m'. Flow parameters corresponding to the sea-water having different dust concentrations are given in Table 4.

Friction factors for pseudo-plastic conditions obtained from the Moody diagram through the curve for smooth pipes are given in Table 5.

V   SELECTION OF POSSIBLE FLOW BEHAVIOUR FOR PIPE DESIGN.

Combination of Bingham plastic and pseudo-plastic behaviours were observed for the concentrations of 200, 167 and 143 g/l. It is obvious from the related curves (Figs. 5 and 6) that for the above mentioned concentrations pseudo-plasticity exists at low flow rates, but for decreasing concentrations ($n < 1/7$, 143 g/l) the influence of pseudo-plasticity is deminishing. It seems from the viscometric data that for concentrations over 125 g/l the rate of flow is responsible for the change in the behaviour of flow.

5 – Behn, V.C., Experimental Determination of Sludge Flow Parameters, ASCE, Vol. 88, No. SA3, May 1962, p. 42.

This investigation had to be undertaken according to the previously determined flow velocities and pipe diameters. The influence of flow rate on the behaviour of flow is obvious from the results, and the selection of the possible flow behaviour corresponding to the flow velocities and pipe diameters in question was necessary.

Because shear rate is a function of flow velocity in the pipe and the diameter of pipe, a selection process was undertaken through the shear rate relationship.

Shear rate at the pipe wall [6]:

$$\left(-\frac{du}{dr}\right)_w = \frac{8U}{D}\left(\frac{1}{1 - \frac{4}{3}\frac{\tau_y}{\tau_w} + \frac{1}{3}\left(\frac{\tau_y}{\tau_w}\right)^4}\right) - \frac{\tau_y}{\eta} \quad \text{(Bingham plastic)} \quad . . (6)$$

$$\left(-\frac{du}{dr}\right)_w = \frac{8U}{D}\left(\frac{3m+1}{4m}\right) . . . . \text{(pseudo-plastic)} \quad . . (7)$$

where

τ_w = shear stress at the pipe wall, and other terms are as previously defined.

Max. and min. shear rate values are given in Table 6. In calculating shear rate at the pipe wall for Bingham plastic behaviour the value of, $\tau_y/\tau_w < 0.4$, in all cases, made it possible to neglect the last term in brackets in Eq. 6. The application of data given in Table 6 to the related flow curves (Figs. 5 and 6) show that flow behaviour for the given flow velocities and pipe diameters is pseudo-plastic.

VI DISCUSSION.

This investigation shows that the flow behaviour determination of sea-water mixed with varying concentrations of CaSi-dust does not give a direct proportionality between shear stress and shear rate under laminar flow conditions, but has to be evaluated from very accurate viscometric data.

6 - Behn, V.D., Derivation of Flow Equations for Sewage Sludges, ASCE, Vol. 86, No. SA6, November 1960, p. 64, 67.

The range of viscometric data used during this investigation was not wide enough for the accurate determination of flow parameters and thixotropy if it existed. As experienced by Behn [5] it is possible to obtain a wider range of viscometric data by using capilary viscometer through which the direct scale-up for pipe-line design is possible.

Settling and non-uniform distribution of dust particles may be the reason for divergence from the Bingham plastic behaviour at low flow rates. However, existing data are not sufficiently extensive to verify this concept.

The economic consequense of this may be quite considerable as shown in the following example:

Friction factor used during the previous investigation for sea-water mixed with FeSi-dust, n = 143 g/l (1/7), flow velocity and pipediameter of 1.1 m/sec. and 150 mm $\emptyset$ respectively was, λ = 0.019. The same value can be used in the case of Bingham-plastic flow for sea-water mixed with CaSi-dust n = 143 g/l (1/7) under the same conditions. Friction loss for both above mentioned situations will be 0.008 m for 1.00 m pipe length. Sea-water mixed with CaSi-dust for n = 143 g/l (1/7) for pseudo-plastic flow gives a friction factor of, $\lambda \approx 0.028$, for which friction loss in 1.00 m pipe length will be 0.0113 m. Under above given conditions the difference in friction loss in a pipe length of 2,000 m for Bingham plastic and pseudo-plastic flow will be 22.6 - 16.0 = 6,6 m.

VII CONCLUSION.

From the teoretical treastise of data based on viscometric measurements on sea-water containing concentrations of CaSi-dust varying between 100 and 200 g/l, the following flow characteristics of the liquid have been evaluated:

1. A combination of Bingham plastic and pseudo-plastic behaviours were observed. For concentrations higher than 125 g/l pseudo-plasticity predominates at low flow rates. For concentrations less than 143 g/l, the influence of pseudo-plasticity is diminishing.

2. Within the given ranges of flow velocity and pipe diameter, the flow
    behaviour of the liquid having concentrations of CaSi-dust over
    125 g/l must be characterized as pseudo-plastic.

3. For concentrations over 125 g/l, friction factors given in Table 5
    are recommended to use for design purposes.

TABLE 1.

Specific weight of sea-water for different dust concentrations.

Dust concentration (n)	Sp. weight (γ)	
	mixed with (Si-met.)	mixed with (CaSi)
1/1 (1.000 g/1)	1.439	1.455
1/2 (500 g/1)	1.232	1.240
1/3 (333 g/1)	1.163	1.168
1/4 (250 g/1)	1.128	1.132
1/5 (200 g/1)	1.108	1.111
1/6 (167 g/1)	1.092	1.097
1/7 (143 g/1)	1.084	1.086
1/8 (125 g/1)	1.077	1.080
1/9 (111 g/1)	1.071	1.073
1/10(100 g/1)	1.066	1.068

TABLE 2.

Coefficient of rigidity for sea-water consisting
of different dust concentrations.

n	1/5(200 g/1)	1/6(167 g/1)	1/7(143 g/1)	1/8(125 g/1)	1/9(111 g/1)
η	0.032	0.026	0.025	0.025	0.023

TABLE 3.

Friction factors obtained from Moody diagram (see Fig. 10), Bingham plastic flow.

D = 80 mm ø D1=100 " D2=120 " D3=150 " U m/sec.	Re for η = 0.032				λ				Re for η = 0.026 Re for η = 0.025 Re for η = 0.023				λ			
	D_1	D_2	D_3	D_4	D_1	D_2	D_3	D_4	D_1	D_2	D_3	D_4	D_1	D_2	D_3	D_4
0.80	25.3×10^3	27.7×10^3	33.3×10^3	41.6×10^3	0.026	0.025	0.023	0.022	32.4×10^3	35.5×10^3	42.6×10^3	50.8×10^3	0.024	0.023	0.022	0.021
1.00	27.7×10^3	34.7×10^3	41.6×10^3	52.0×10^3	0.025	0.023	0.022	0.021	35.5×10^3	44.4×10^3	53.3×10^3	53.5×10^3	0.023	0.022	0.020	0.020
1.20	33.3×10^3	41.5×10^3	49.9×10^3	62.4×10^3	0.023	0.022	0.021	0.020	42.6×10^3	53.3×10^3	63.9×10^3	76.2×10^3	0.022	0.020	0.020	0.019

TABLE 4.

Flow parameters for sea-water having different dust concentrations.

n	1/5 (200 g/l)	1/6 (157 g/l)	1/7 (143 g/l)	1/8 (125 g/l)	1/9 (111 g/l)
n=m´	0.032	0.27	0.27	Bingham plastic	Bingham plastic
k	3.6	3.4	3.4		
k´	4.1	3.9	3.9		

TABLE 5.

Friction factors obtained from Moody diagram, pseudo-plastic flow.

D_1 = 80 mm Ø
D_2 = 100 "
D_3 = 120 "
D_4 = 150 "

U m/sec.	Re for k' = 4.1, m' = 0.32				λ				Re for k' = 3.9, m' = 0.27, 0.26, 0.23				λ			
	D_1	D_2	D_3	D_4	D_1	D_2	D_3	D_4	D_1	D_2	D_3	D_4	D_1	D_2	D_3	D_4
0.80	2882	3150	3307	4316	0.044	0.043	0.042	0.039	3823	3958	4346	4500	0.041	0.040	0.039	0.037
1.00	4910	4580	4809	6275	0.038	0.037	0.036	0.036	5693	5896	6474	6704	0.037	0.036	0.036	0.034
1.20	5983	6540	6867	8960	0.036	0.035	0.034	0.033	7821	8099	8893	9210	0.033	0.032	0.032	0.032

TABLE 6.

Max. and min. shear rate values for the selection of flow behaviour.

U m/sec.	Pseudo-plastic				Bingham plastic	
	m = 0.32 (du/dr)$_w$		m = 0.27 (du/dr)$_w$		(du/dr)$_w$	
	D_1(max.)	D_4(min.)	D_1(max.)	D_4(min.)	D_1(max.)	D_4(min.)
0.80	-	64	-	69	-	43
1.20	184	-	202	-	120	-

Fig. 1 Specific weight and consentration relationship.

Specific weight of sea water
mixed with dust   : γ

Sp. w. of sea water   : γ_s = 1.025

Sp. w. of dust   : γ dust = 1.748 (Si-met) , 1.795 (CaSi)

Dust consentration   : $n = \dfrac{1\,kg\ dust}{sea\ w.\ mixed\ with\ dust}$

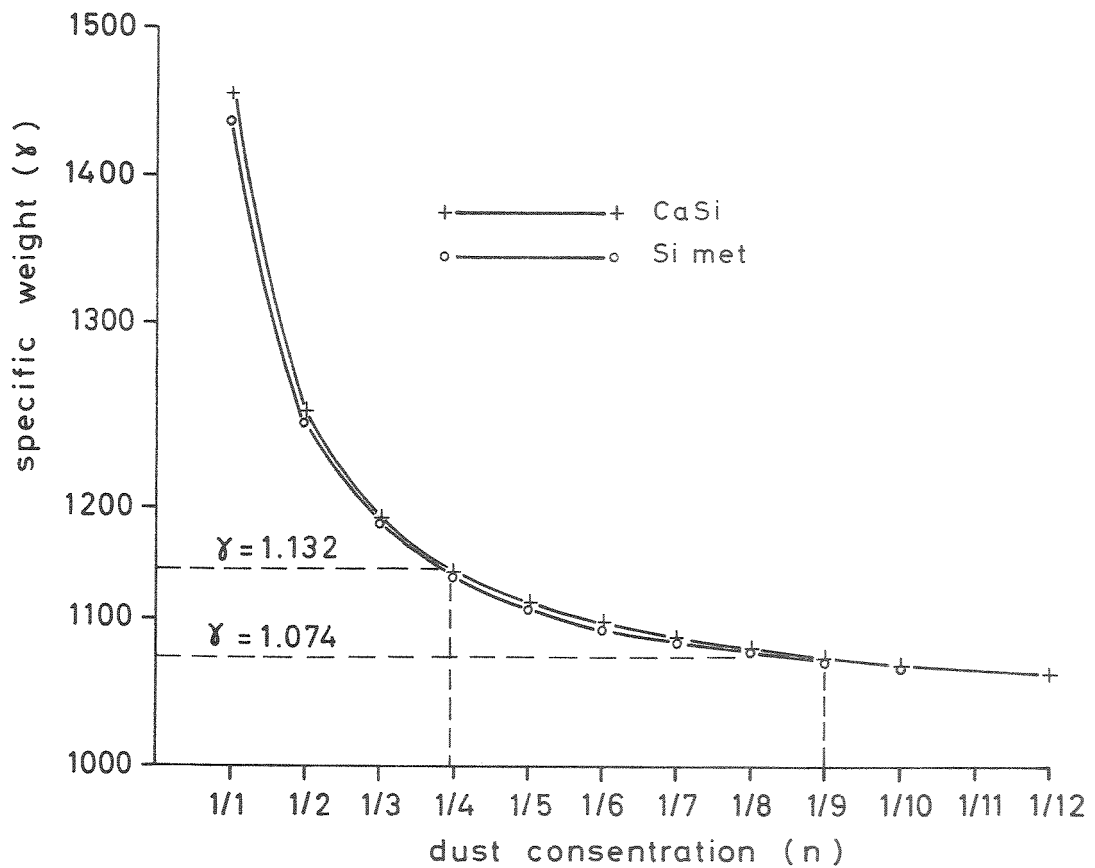$$\gamma = n \left[1 + \gamma_s \left(\frac{1}{n} - \frac{1}{\gamma\,dust} \right) \right]$$
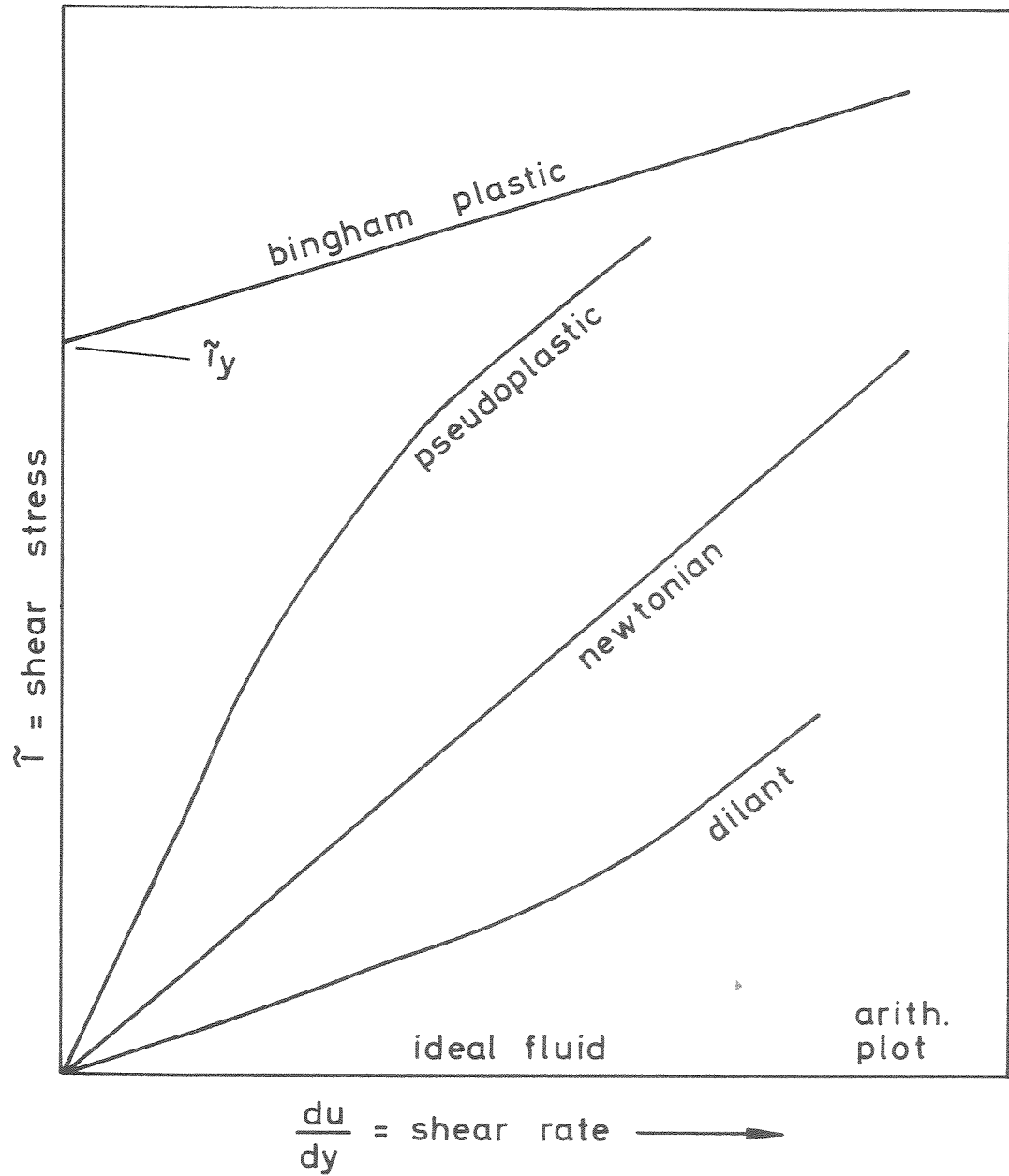
Fig. 2   Fluid   flow   curves.

Fig. 2   Fluid   flow   curves.

Fig. 3 Rotational viscometer data
(sea water mixed with CaSi-dust)

shear rate, du/dy, seconds^{-1}

shear stress, ℓ, in dynes per centimeter square

(143 g/l)

(200 g/l)

(166.6 g/l)
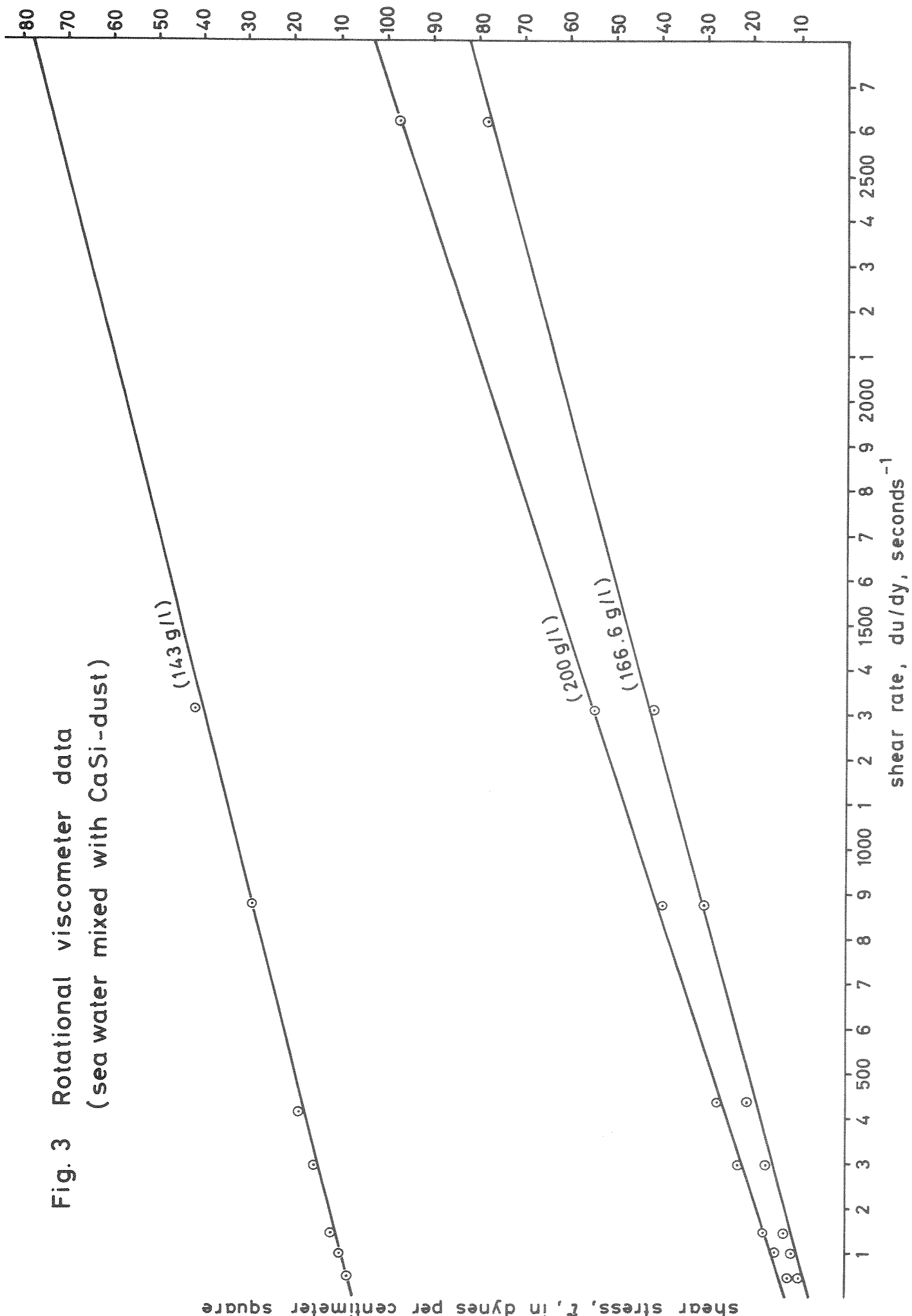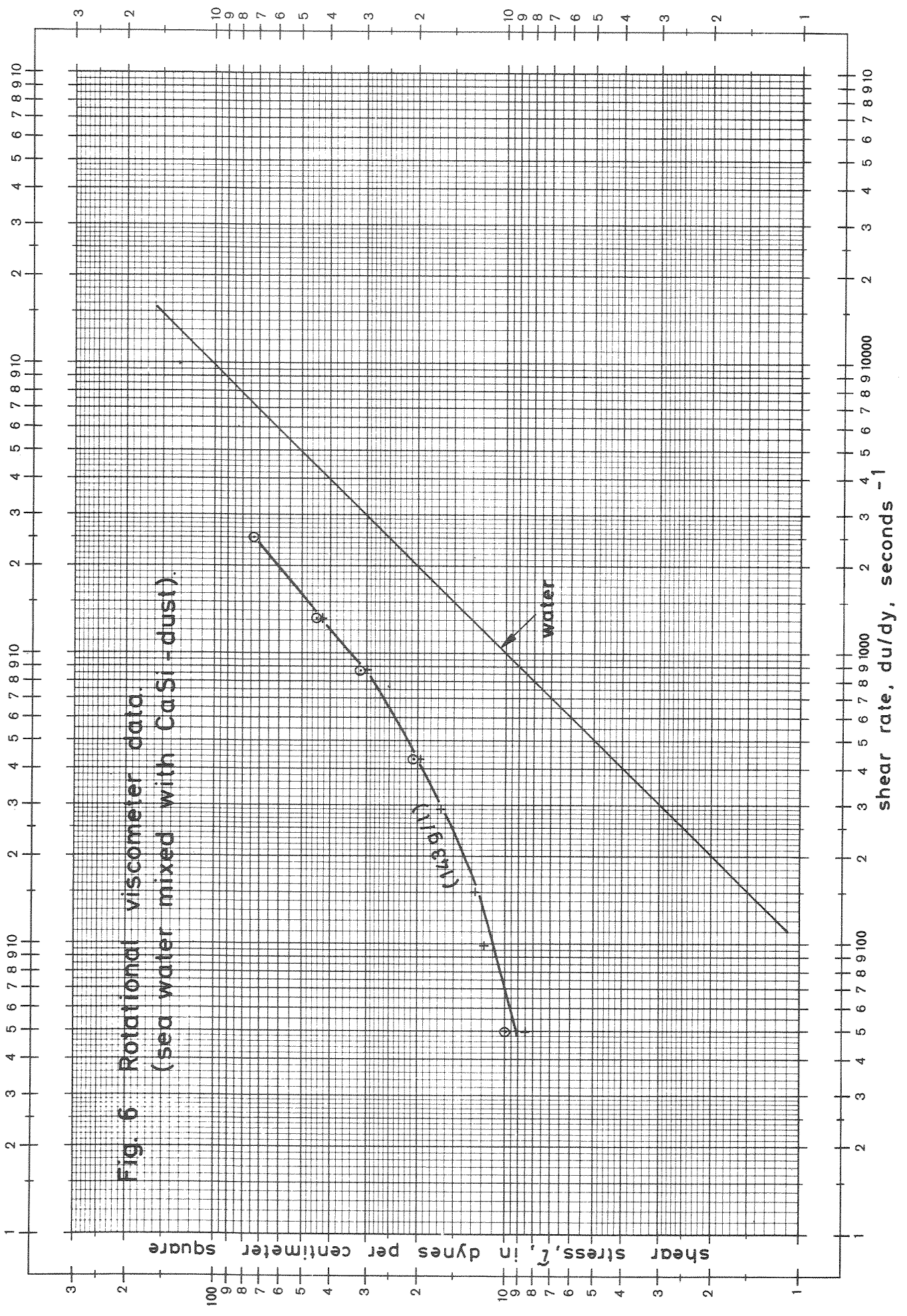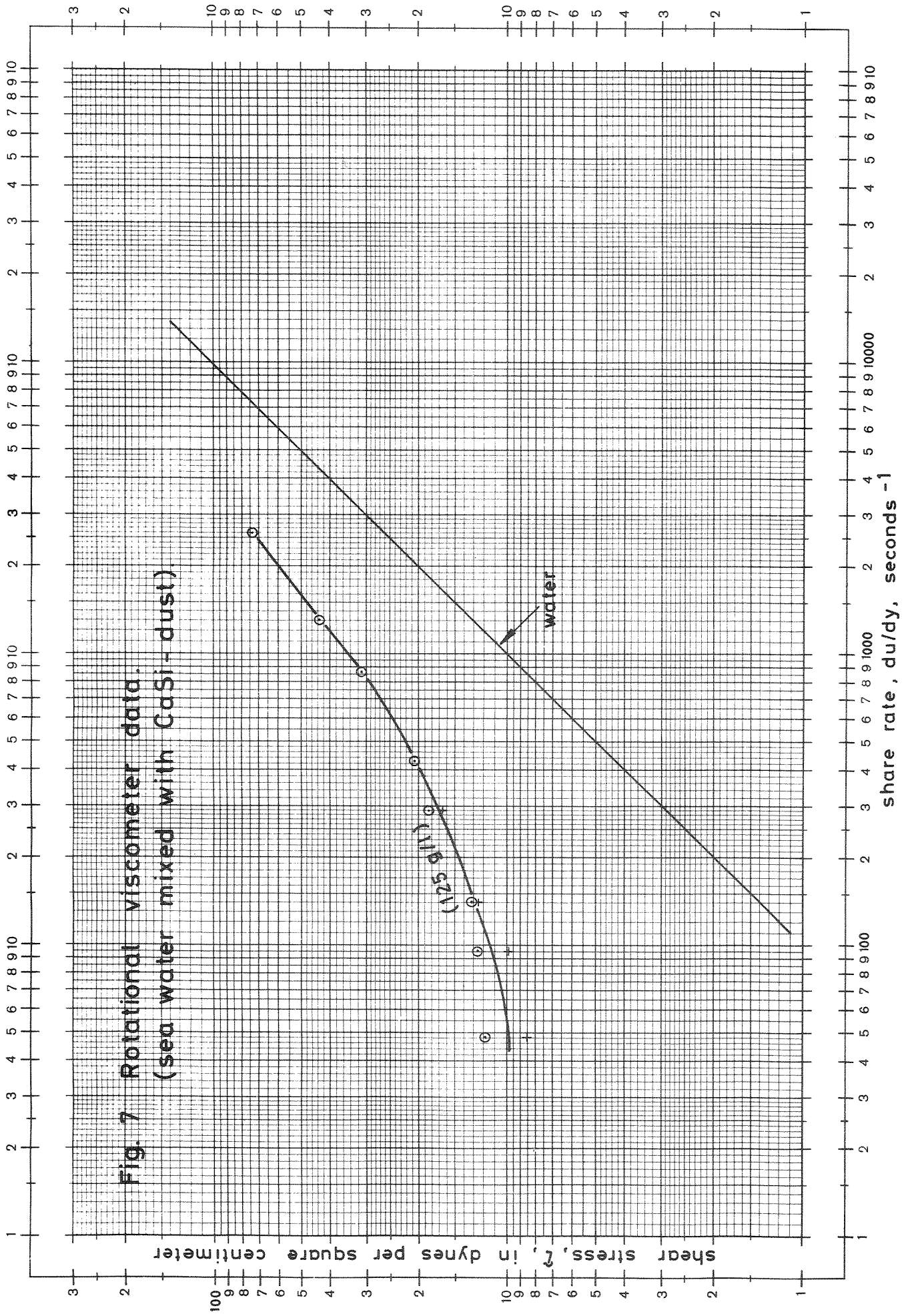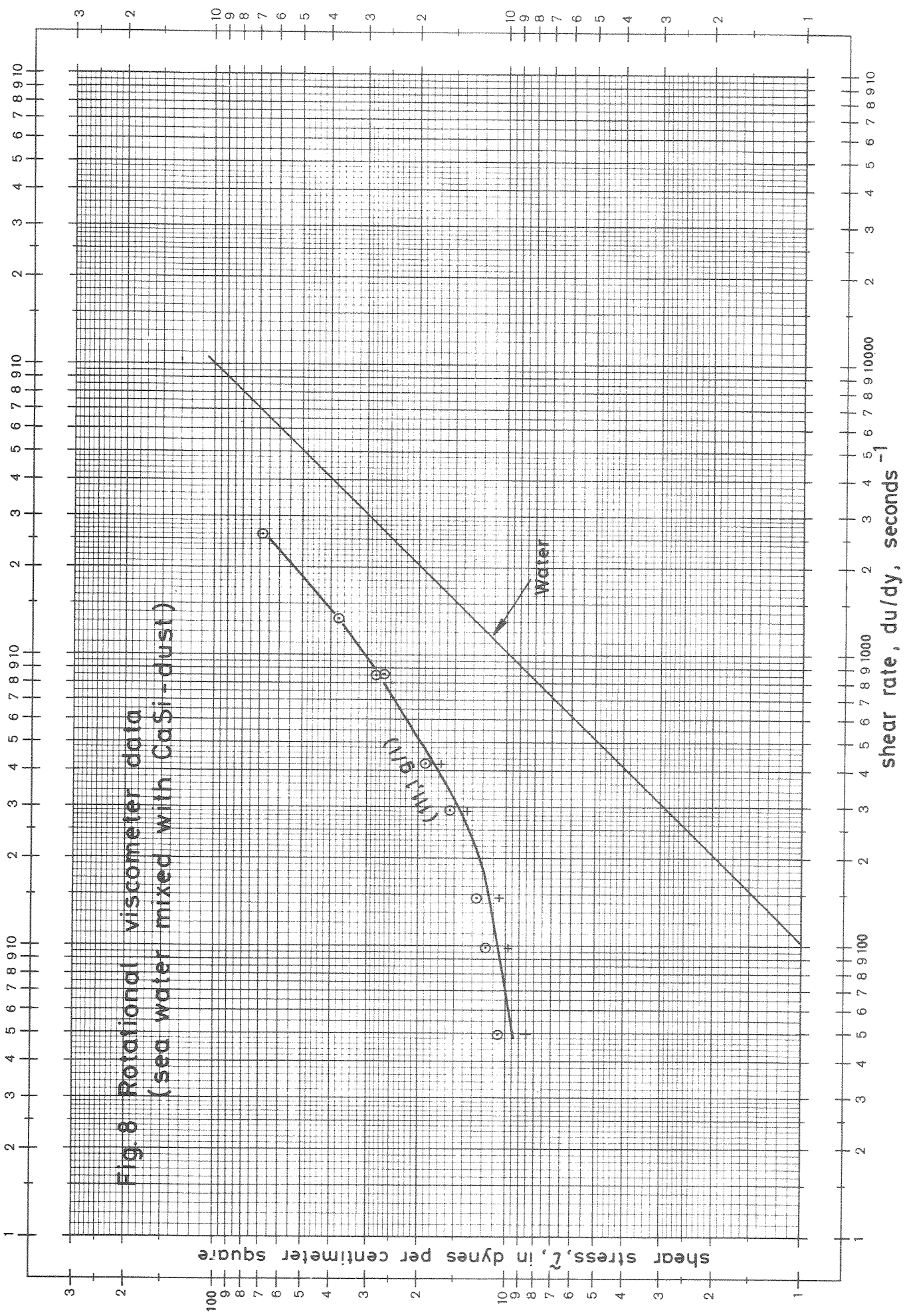
Fig. 4   Rotational viscometer data
(sea water mixed with CaSi-dust)
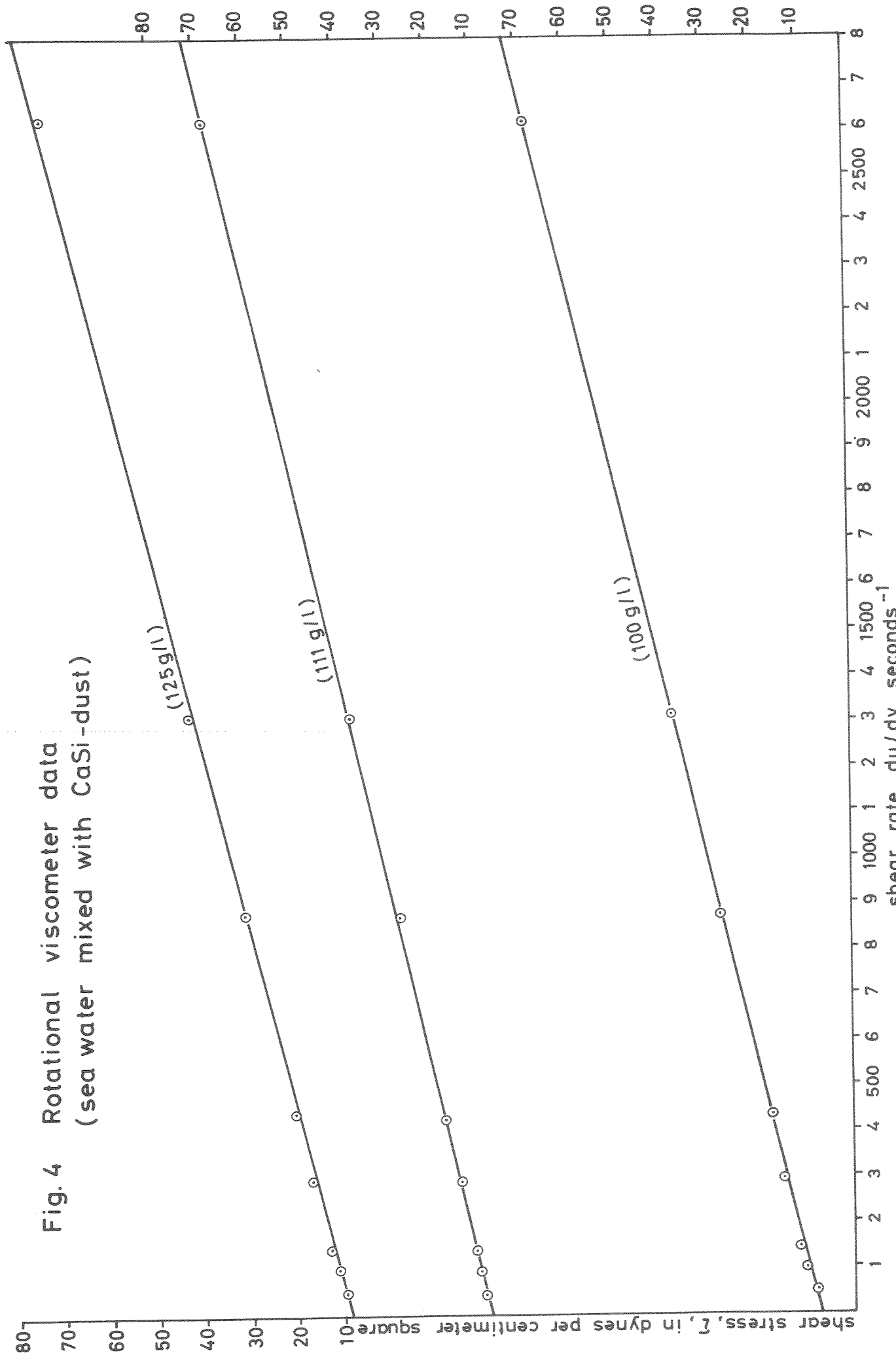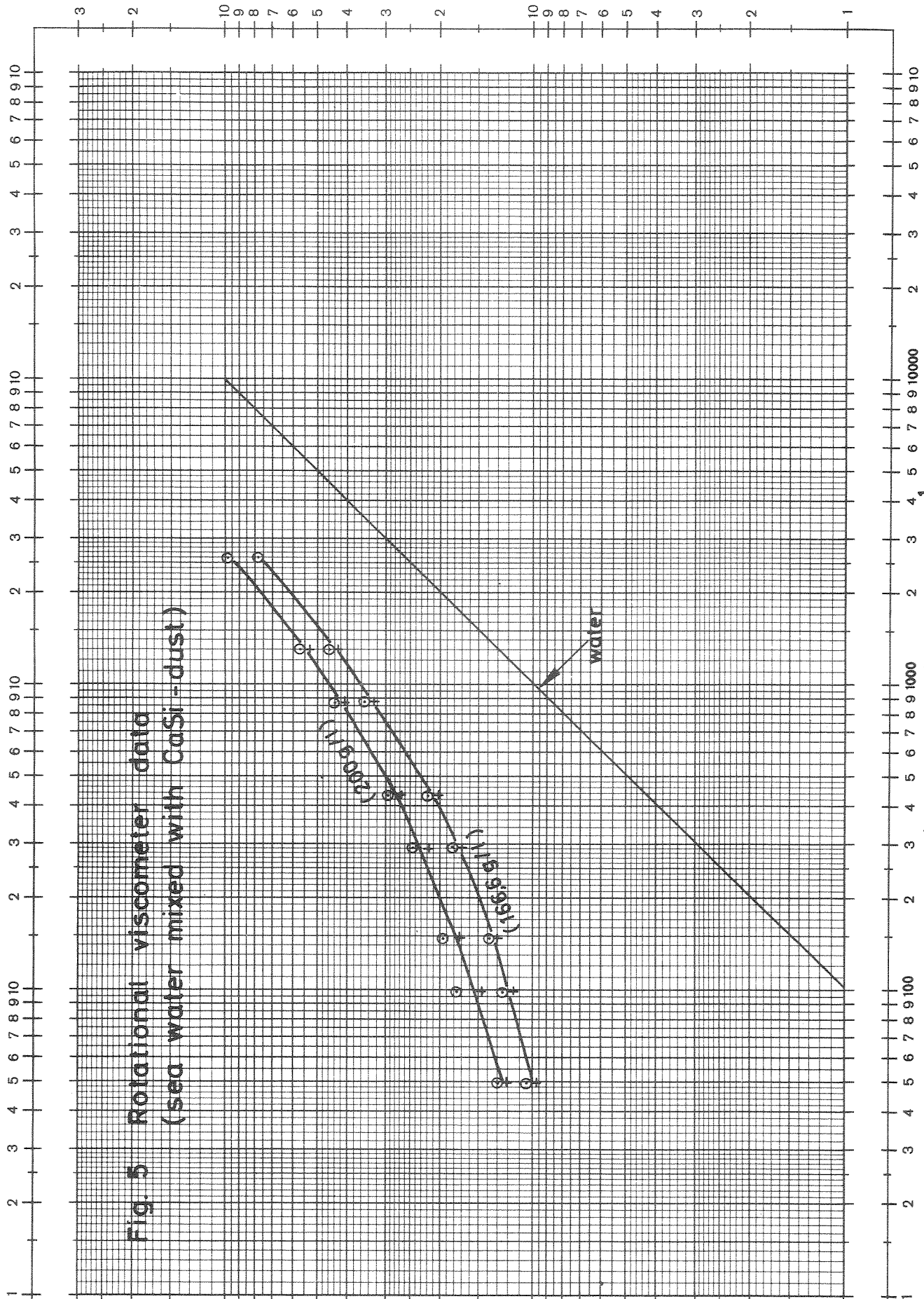
Fig. 5 Rotational viscometer data
(sea water mixed with CaSi–dust)

shear rate, du/dy, seconds⁻¹

shear stress, t, in dynes per centimeter square

Fig. 6 Rotational viscometer data.
(sea water mixed with CaSi-dust).

Fig. 7 Rotational viscometer data.
(sea water mixed with CaSi-dust)

shear stress, f, in dynes per square centimeter

share rate, du/dy, seconds $^{-1}$

water

(125 l/l)

Fig. 8 Rotational viscometer data
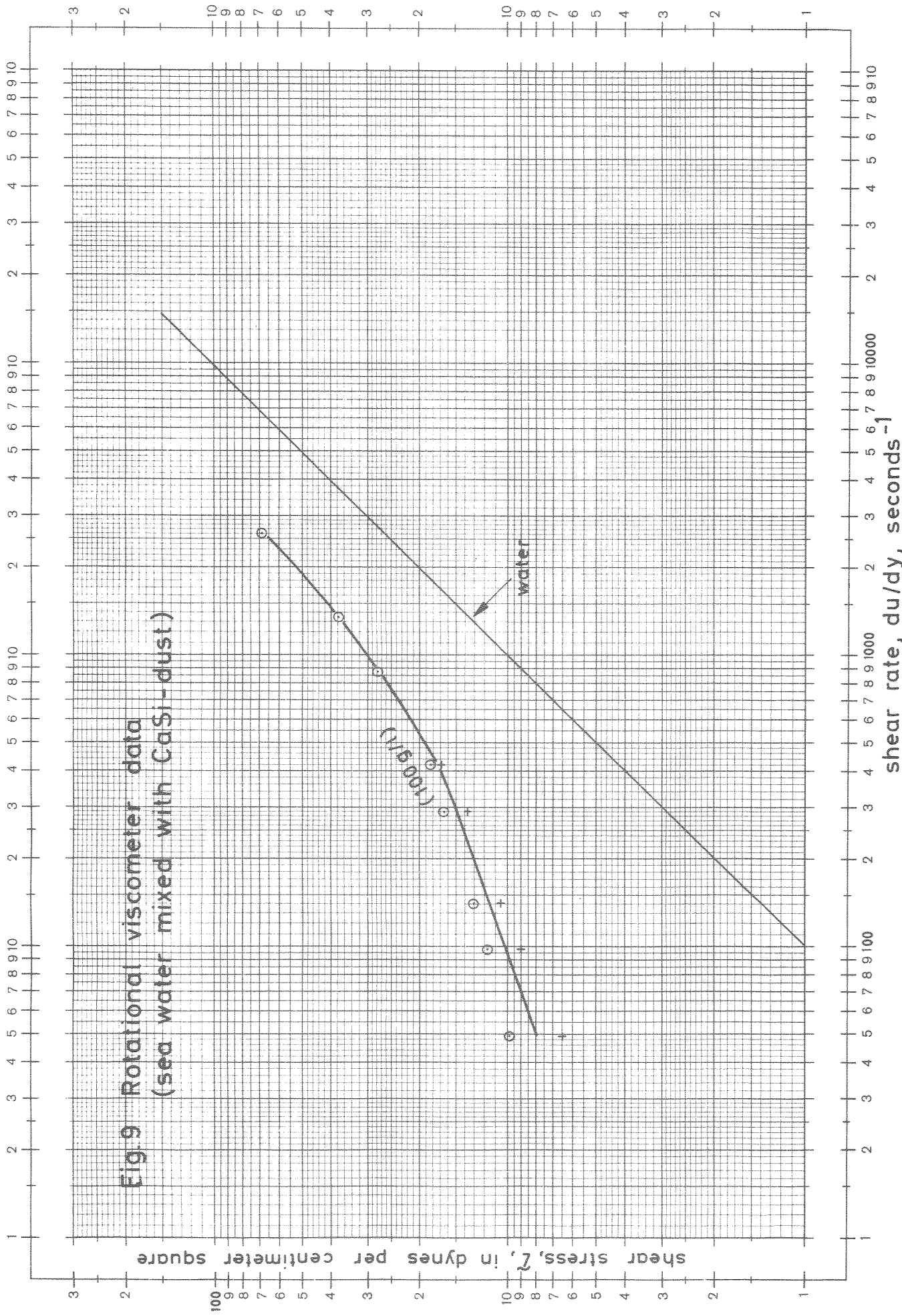(sea water mixed with CaSi-dust)
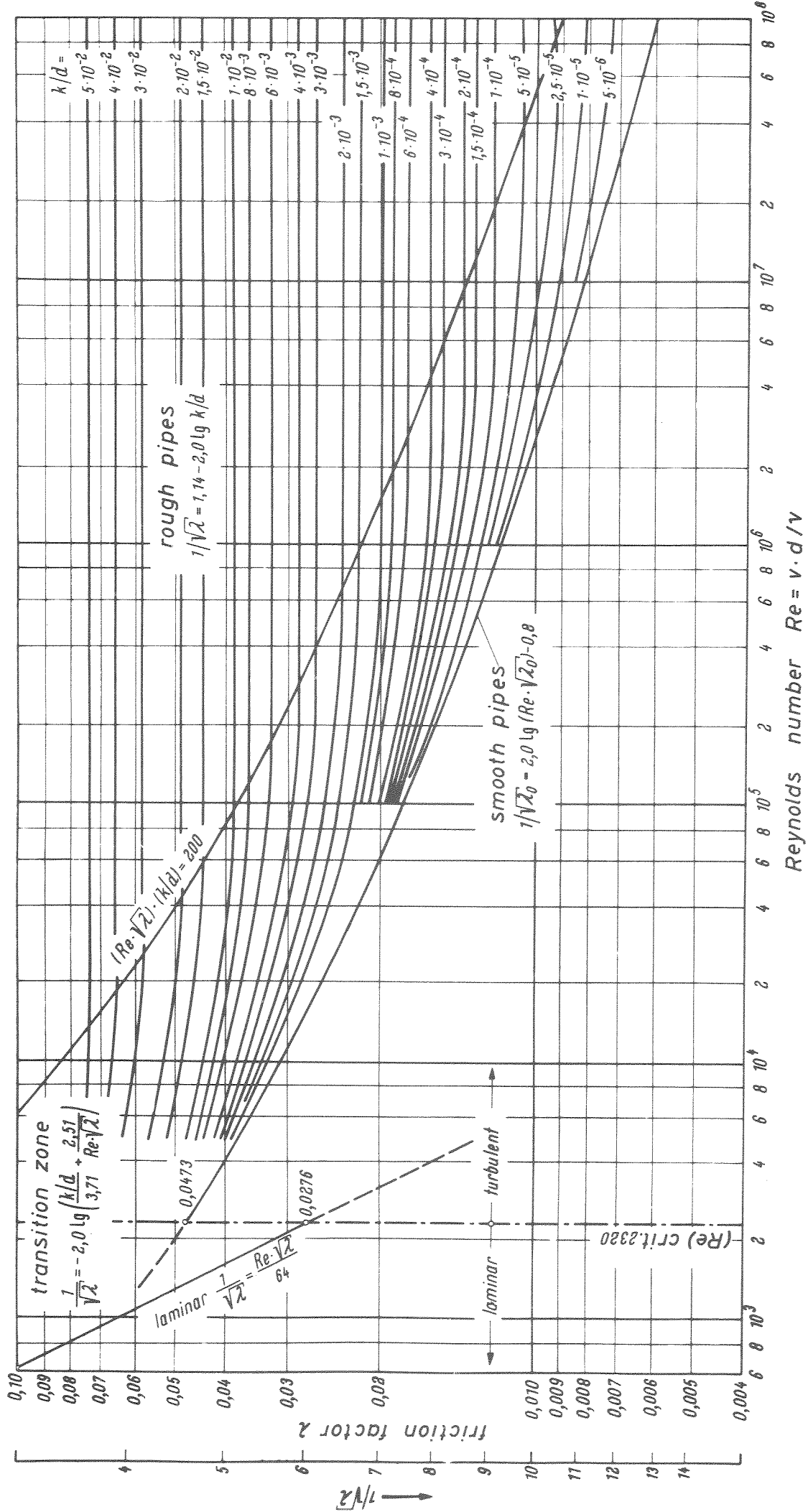
Fig. 9   Rotational viscometer data (sea water mixed with CaSi-dust)

Fig. 10 Moody Diagram for friction in pipes.